UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS General Certificate of Education Advanced Subsidiary Level

## MATHEMATICS

9709/22
Paper 2 Pure Mathematics 2 (P2)

| Additional Materials: | Answer Booklet/Paper <br> Graph Paper <br> List of Formulae (MF9) |
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## READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.
At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [] at the end of each question or part question.
The total number of marks for this paper is 50 .
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

1
(i) Find $\int \frac{2}{4 x-1} \mathrm{~d} x$.
(ii) Hence find $\int_{1}^{7} \frac{2}{4 x-1} \mathrm{~d} x$, expressing your answer in the form $\ln a$, where $a$ is an integer.

2 The curve $y=\frac{\mathrm{e}^{3 x-1}}{2 x}$ has one stationary point. Find the coordinates of this stationary point.

3 Solve the equation $2 \cot ^{2} \theta-5 \operatorname{cosec} \theta=10$, giving all solutions in the interval $0^{\circ} \leqslant \theta \leqslant 360^{\circ}$.

4 (i) The polynomial $a x^{3}+b x^{2}-25 x-6$, where $a$ and $b$ are constants, is denoted by $\mathrm{p}(x)$. It is given that $(x-3)$ and $(x+2)$ are factors of $\mathrm{p}(x)$. Find the values of $a$ and $b$.
(ii) When $a$ and $b$ have these values, factorise $\mathrm{p}(x)$ completely.

5 The parametric equations of a curve are

$$
x=1+\sqrt{ } t, \quad y=3 \ln t
$$

(i) Find the exact value of the gradient of the curve at the point $P$ where $y=6$.
(ii) Show that the tangent to the curve at $P$ passes through the point $(1,0)$.

6 (a) Find $\int(\sin x-\cos x)^{2} d x$.
(b) (i) Use the trapezium rule with 2 intervals to estimate the value of

$$
\int_{\frac{1}{4} \pi}^{\frac{1}{2} \pi} \operatorname{cosec} x \mathrm{~d} x
$$

giving your answer correct to 3 decimal places.
(ii) Using a sketch of the graph of $y=\operatorname{cosec} x$ for $0<x \leqslant \frac{1}{2} \pi$, explain whether the trapezium rule gives an under-estimate or an over-estimate of the true value of the integral in part (i).


The diagram shows part of the curve $y=8 x+\frac{1}{2} \mathrm{e}^{x}$. The shaded region $R$ is bounded by the curve and by the lines $x=0, y=0$ and $x=a$, where $a$ is positive. The area of $R$ is equal to $\frac{1}{2}$.
(i) Find an equation satisfied by $a$, and show that the equation can be written in the form

$$
\begin{equation*}
a=\sqrt{ }\left(\frac{2-\mathrm{e}^{a}}{8}\right) \tag{5}
\end{equation*}
$$


(iii) Use the iterative formula $a_{n+1}=\sqrt{ }\left(\frac{2-\mathrm{e}^{a_{n}}}{8}\right)$ to determine this root correct to 2 decimal places. Give the result of each iteration to 4 decimal places.

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