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UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Subsidiary Level and GCE Advanced Level

MARK SCHEME for the October/November 2011 question paper for the guidance of teachers

9709 MATHEMATICS

9709/32

Paper 3, maximum raw mark 75

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes must be read in conjunction with the question papers and the report on the examination.

• Cambridge will not enter into discussions or correspondence in connection with these mark schemes.

Cambridge is publishing the mark schemes for the October/November 2011 question papers for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level syllabuses and some Ordinary Level syllabuses.

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Mark Scheme Notes

Marks are of the following three types:

- M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B Mark for a correct result or statement independent of method marks.
- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.
- Note: B2 or A2 means that the candidate can earn 2 or 0.
 B2/1/0 means that the candidate can earn anything from 0 to 2.

The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.
- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

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The following abbreviations may be used in a mark scheme or used on the scripts:

| AEF | Any Equivalent Form (of answer is equally acceptable) |
|-----|---|
| AG | Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid) |
| BOD | Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear) |
| CAO | Correct Answer Only (emphasising that no "follow through" from a previous error is allowed) |
| CWO | Correct Working Only – often written by a 'fortuitous' answer |
| ISW | Ignore Subsequent Working |
| MR | Misread |
| PA | Premature Approximation (resulting in basically correct work that is insufficiently accurate) |
| sos | See Other Solution (the candidate makes a better attempt at the same question) |
| SR | Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance) |

Penalties

- MR −1 A penalty of MR −1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures this is regarded as an error in accuracy. An MR −2 penalty may be applied in particular cases if agreed at the coordination meeting.
- PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

| 1 Rearrange as $e^{3x} - e^{2} - 6 = 0$, or $u^2 - u - 6 = 0$, or equivalent Solve a 3-term quadratic for e^x or for u Obtain simplified solution $e^x = 3$ or $u = 3$ Obtain final answer $x = 1.10$ and no other 2 EITHER: Use chain rule obtain $\frac{dx}{dt} = 6 \sin t \cos t$, or equivalent obtain $\frac{dx}{dt} = 6 \sin t \cos t$, or equivalent Use $\frac{dy}{dt} = \frac{dy}{dt} + \frac{dx}{dt}$ Obtain final answer $\frac{dy}{dt} = -\cos t$ OR: Express y in terms of x and use chain rule Obtain $\frac{dy}{dx} = k(2 - \frac{x}{3})^{\frac{1}{2}}$ or equivalent Obtain $\frac{dy}{dx} = k(2 - \frac{x}{3})^{\frac{1}{2}}$ or equivalent Obtain $\frac{dy}{dx} = -(2 - \frac{x}{3})^{\frac{1}{2}}$ or equivalent Obtain $\frac{dy}{dx} = -(2 - \frac{x}{3})^{\frac{1}{2}}$ or equivalent Express y in terms of x and use chain rule Obtain $\frac{dy}{dx} = -(2 - \frac{x}{3})^{\frac{1}{2}}$ or equivalent Express derivative in terms of t Obtain inal answer $\frac{dy}{dx} = -\cos t$ A1 [5] 3 (i) EITHER: Attempt division by $x^2 - x + 1$ reaching a partial quotient of $x^2 + kx$ M1 Obtain quotient $x^2 + 4x + 3$ Equate remainder of form t to zero and solve for a , or equivalent Obtain answer $a = 1$ Obtain in $a + 1$ Obta | | Page 4 | Mark Scheme: Teachers' version | Syllabus | Paper | , |
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| Obtain $\frac{dy}{dx} = -(2 - \frac{x}{3})^{\frac{1}{2}}$, or equivalent Express derivative in terms of t Obtain final answer $\frac{dy}{dx} = -\cos t$ A1 [5] 3 (i) EITHER: Attempt division by $x^2 - x + 1$ reaching a partial quotient of $x^2 + kx$ Obtain quotient $x^2 + 4x + 3$ Equate remainder of form k to zero and solve for a , or equivalent Obtain answer $a = 1$ OR: Substitute a complex zero of $x^2 - x + 1$ in $p(x)$ and equate to zero M1 Obtain a correct equation in a in any unsimplified form Expand terms, use $t^2 = -1$ and solve for a Obtain answer $a = 1$ [4] [5] 18 [SR: The first M1 is earned if inspection reaches an unknown factor $x^2 + Bx + C$ and an equation in B and/or C , or an unknown factor $Ax^2 + Bx + 3$ and an equation in A and/or B . The second M1 is only earned if use of the equation $a = B - C$ is seen or implied.] (ii) State answer, e.g. $x = -3$ State answer, e.g. $x = -1$ and no others B1 [2] 4 Separate variables and attempt integration of at least one side Obtain term $\ln(x + 1)$ Obtain correct term $\frac{1}{2} \ln \sin 2\theta$ Evaluate a constant, or use limits $\theta = \frac{1}{12}\pi$, $x = 0$ in a solution containing terms $a \ln(x + 1)$ and $b \ln \sin 2\theta$ Obtain solution in any form, e.g. $\ln(x + 1) = \frac{1}{2} \ln \sin 2\theta - \frac{1}{2} \ln \frac{1}{2}$ (f.t. on $k = \pm 1, \pm 2$, or $\pm \frac{1}{2}$) A1 $\sqrt{\frac{1}{2}}$ | | | 1 | | | |
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| Express derivative in terms of t Obtain final answer $\frac{dy}{dx} = -\cos t$ A1 [5] 3 (i) EITHER: Attempt division by $x^2 - x + 1$ reaching a partial quotient of $x^2 + kx$ M1 Obtain quotient $x^2 + 4x + 3$ Equate remainder of form kx to zero and solve for a , or equivalent Obtain answer $a = 1$ OR: Substitute a complex zero of $x^2 - x + 1$ in $p(x)$ and equate to zero M1 Obtain a correct equation in a in any unsimplified form Expand terms, use $i^2 = -1$ and solve for a Obtain answer $a = 1$ [SR: The first M1 is earned if inspection reaches an unknown factor $x^2 + Bx + C$ and an equation in B and/or C , or an unknown factor $Ax^2 + Bx + 3$ and an equation in A and/or B . The second M1 is only earned if use of the equation $a = B - C$ is seen or implied.] (ii) State answer, e.g. $x = -3$ State answer, e.g. $x = -1$ and no others B1 [2] 4 Separate variables and attempt integration of at least one side Obtain term k ln $\sin 2\theta$, where $k = \pm 1, \pm 2$, or $\pm \frac{1}{2}$ Obtain correct term $\frac{1}{2} \ln \sin 2\theta$ Evaluate a constant, or use limits $\theta = \frac{1}{12}\pi$, $x = 0$ in a solution containing terms $a \ln(x + 1)$ and $b \ln \sin 2\theta$ Obtain solution in any form, e.g. $\ln(x + 1) = \frac{1}{2} \ln \sin 2\theta - \frac{1}{2} \ln \frac{1}{2}$ (f.t. on $k = \pm 1, \pm 2$, or $\pm \frac{1}{2}$) A1 $\sqrt{\frac{1}{2}}$ | | (| Obtain $\frac{dy}{dx} = -(2 - \frac{x}{3})^{\frac{1}{2}}$, or equivalent | | A1 | |
| 3 (i) <i>EITHER</i> : Attempt division by $x^2 - x + 1$ reaching a partial quotient of $x^2 + kx$ M1 Obtain quotient $x^2 + 4x + 3$ A1 Equate remainder of form k to zero and solve for a , or equivalent M1 Obtain answer $a = 1$ A1 OR: Substitute a complex zero of $x^2 - x + 1$ in $p(x)$ and equate to zero M1 Obtain a correct equation in a in any unsimplified form A1 Expand terms, use $i^2 = -1$ and solve for a M1 Obtain answer $a = 1$ [SR: The first M1 is earned if inspection reaches an unknown factor $x^2 + Bx + C$ and an equation in B and/or C , or an unknown factor $Ax^2 + Bx + 3$ and an equation in A and/or B . The second M1 is only earned if use of the equation $a = B - C$ is seen or implied.] (ii) State answer, e.g. $x = -3$ B1 State answer, e.g. $x = -1$ and no others B1 4 Separate variables and attempt integration of at least one side Obtain term $\ln(x + 1)$ A1 Obtain correct term $\frac{1}{2} \ln \sin 2\theta$ A1 Evaluate a constant, or use limits $\theta = \frac{1}{12}\pi$, $x = 0$ in a solution containing terms $a \ln(x + 1)$ and $b \ln \sin 2\theta$ M1 Obtain solution in any form, e.g. $\ln(x + 1) = \frac{1}{2} \ln \sin 2\theta - \frac{1}{2} \ln \frac{1}{2}$ (f.t. on $k = \pm 1, \pm 2$, or $\pm \frac{1}{2}$) A1 $\sqrt{\frac{1}{2}}$ | | I | Express derivative in terms of t | | M1 | |
| Obtain quotient $x^2 + 4x + 3$ Equate remainder of form lx to zero and solve for a , or equivalent lx M1 Obtain answer $a = 1$ A1 OR: Substitute a complex zero of $x^2 - x + 1$ in $p(x)$ and equate to zero lx M1 Obtain a correct equation in a in any unsimplified form A1 Expand terms, use $i^2 = -1$ and solve for a M1 Obtain answer $a = 1$ A1 [4] [5R: The first M1 is earned if inspection reaches an unknown factor $x^2 + Bx + C$ and an equation in a and/or a 0. The second M1 is only earned if use of the equation $a = B - C$ 1 is seen or implied.] (ii) State answer, e.g. $a = -3$ B1 State answer, e.g. $a = -1$ and no others B1 4 Separate variables and attempt integration of at least one side Obtain term $a = b = b$ 0. Where $a = b$ 1 M1 Obtain correct term $a = b$ 2 M1 Obtain correct term $a = b$ 3 M2 Evaluate a constant, or use limits $a = b$ 4 M1 Obtain solution in any form, e.g. $a = b$ 6 in a solution containing terms $a = b$ 6 M1 Obtain solution in any form, e.g. $a = b$ 6 in a solution containing terms $a = b$ 7 M1 Obtain solution in any form, e.g. $a = b$ 7 M1 Obtain solution in any form, e.g. $a = b$ 8 M1 Obtain solution in any form, e.g. $a = b$ 9 N1 Obtain solution in any form, e.g. $a = b$ 9 N1 Obtain solution in any form, e.g. $a = b$ 9 N1 Obtain solution in any form, e.g. $a = b$ 9 N1 Obtain solution in any form, e.g. $a = b$ 9 N1 Obtain solution in any form, e.g. $a = b$ 9 N1 Obtain solution in any form, e.g. $a = b$ 9 N1 Obtain solution in any form, e.g. $a = b$ 9 N1 Obtain solution in any form, e.g. $a = b$ 9 N1 Obtain solution in any form, e.g. $a = b$ 9 N1 Obtain solution in any form, e.g. $a = b$ 9 N1 Obtain solution in any form, e.g. $a = b$ 9 N1 Obtain solution in any form, e.g. $a = b$ 9 N1 Obtain solution in any form, e.g. $a = b$ 9 N1 Obtain solution in any form, e.g. $a = b$ 9 N1 Obtain solution in any form, e.g. $a = b$ 9 N1 Obtain solution in any form, e.g. $a = b$ 9 N1 Obtain solution in any form, e.g. $a = b$ 9 N1 Obtain solution in any form in any fo | | (| Obtain final answer $\frac{dy}{dx} = -\cos t$ | | A 1 | [5] |
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| Equate remainder of form lx to zero and solve for a , or equivalent Obtain answer $a=1$ OR: Substitute a complex zero of x^2-x+1 in $p(x)$ and equate to zero M1 Obtain a correct equation in a in any unsimplified form Expand terms, use $i^2=-1$ and solve for a Obtain answer $a=1$ [SR: The first M1 is earned if inspection reaches an unknown factor x^2+Bx+C and an equation in B and/or C , or an unknown factor Ax^2+Bx+3 and an equation in A and/or B . The second M1 is only earned if use of the equation $a=B-C$ is seen or implied.] (ii) State answer, e.g. $x=-3$ State answer, e.g. $x=-1$ and no others B1 Obtain term $ln(x+1)$ Obtain term $ln(x+1)$ Obtain term $ln(x+1)$ Obtain correct term $ln(x+1)$ Obtain correct term $ln(x+1)$ Obtain correct term $ln(x+1)$ Obtain correct term $ln(x+1)$ Obtain solution in any form, e.g. $ln(x+1)=\frac{1}{2}ln\sin 2\theta$ A1 Obtain solution in any form, e.g. $ln(x+1)=\frac{1}{2}ln\sin 2\theta-\frac{1}{2}ln\frac{1}{2}$ (f.t. on $k=\pm 1, \pm 2$, or $\pm \frac{1}{2}$) A1 $\sqrt{1}$ | 3 | (I) LITTL | Obtain quotient $x^2 + 4x + 3$ | OI X KX | | |
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| Obtain a correct equation in a in any unsimplified form Expand terms, use $i^2 = -1$ and solve for a Obtain answer $a = 1$ [SR: The first M1 is earned if inspection reaches an unknown factor $x^2 + Bx + C$ and an equation in B and/or C , or an unknown factor $Ax^2 + Bx + 3$ and an equation in A and/or B . The second M1 is only earned if use of the equation $a = B - C$ is seen or implied.] (ii) State answer, e.g. $x = -3$ State answer, e.g. $x = -1$ and no others B1 Separate variables and attempt integration of at least one side Obtain term $\ln(x + 1)$ Obtain term $k \ln \sin 2\theta$, where $k = \pm 1, \pm 2$, or $\pm \frac{1}{2}$ Obtain correct term $\frac{1}{2} \ln \sin 2\theta$ Evaluate a constant, or use limits $\theta = \frac{1}{12}\pi$, $x = 0$ in a solution containing terms $a \ln(x + 1)$ and $b \ln \sin 2\theta$ Obtain solution in any form, e.g. $\ln(x + 1) = \frac{1}{2} \ln \sin 2\theta - \frac{1}{2} \ln \frac{1}{2}$ (f.t. on $k = \pm 1, \pm 2$, or $\pm \frac{1}{2}$) A1 $\sqrt{\frac{1}{2}}$ | | | Obtain answer $a = 1$ | | A 1 | |
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| Obtain answer $a=1$ [SR: The first M1 is earned if inspection reaches an unknown factor $x^2 + Bx + C$ and an equation in B and/or C , or an unknown factor $Ax^2 + Bx + 3$ and an equation in A and/or B . The second M1 is only earned if use of the equation $a = B - C$ is seen or implied.] (ii) State answer, e.g. $x = -3$ B1 State answer, e.g. $x = -1$ and no others B1 Separate variables and attempt integration of at least one side Obtain term $\ln(x + 1)$ A1 Obtain term $k \ln \sin 2\theta$, where $k = \pm 1, \pm 2$, or $\pm \frac{1}{2}$ M1 Obtain correct term $\frac{1}{2} \ln \sin 2\theta$ A1 Evaluate a constant, or use limits $\theta = \frac{1}{12}\pi$, $x = 0$ in a solution containing terms $a \ln(x + 1)$ and $b \ln \sin 2\theta$ M1 Obtain solution in any form, e.g. $\ln(x + 1) = \frac{1}{2} \ln \sin 2\theta - \frac{1}{2} \ln \frac{1}{2}$ (f.t. on $k = \pm 1, \pm 2$, or $\pm \frac{1}{2}$) A1 $\sqrt{1}$ | | | | | | |
| [SR: The first M1 is earned if inspection reaches an unknown factor x² + Bx + C and an equation in B and/or C, or an unknown factor Ax² + Bx + 3 and an equation in A and/or B. The second M1 is only earned if use of the equation a = B - C is seen or implied.] (ii) State answer, e.g. x = -3 State answer, e.g. x = -1 and no others B1 State answer, e.g. x = -1 and no others 4 Separate variables and attempt integration of at least one side Obtain term ln(x + 1) Obtain term k ln sin 2θ, where k = ±1, ±2, or ±½ M1 Obtain correct term ½ ln sin 2θ Evaluate a constant, or use limits θ = ½π, x = 0 in a solution containing terms a ln(x + 1) and b ln sin 2θ Obtain solution in any form, e.g. ln(x + 1) = ½ ln sin 2θ - ½ ln ½ (f.t. on k = ±1, ±2, or ±½) A1√ | | | • | | | F 4 3 |
| State answer, e.g. $x = -1$ and no others B1 [2] 4 Separate variables and attempt integration of at least one side Obtain term $\ln(x+1)$ Obtain term $k \ln \sin 2\theta$, where $k = \pm 1, \pm 2$, or $\pm \frac{1}{2}$ M1 Obtain correct term $\frac{1}{2} \ln \sin 2\theta$ Evaluate a constant, or use limits $\theta = \frac{1}{12}\pi$, $x = 0$ in a solution containing terms $a \ln(x+1)$ and $b \ln \sin 2\theta$ M1 Obtain solution in any form, e.g. $\ln(x+1) = \frac{1}{2} \ln \sin 2\theta - \frac{1}{2} \ln \frac{1}{2}$ (f.t. on $k = \pm 1, \pm 2$, or $\pm \frac{1}{2}$) A1 $\sqrt{\frac{1}{2}}$ | | equation | the first M1 is earned if inspection reaches an unknown factor on in B and/or C, or an unknown factor $Ax^2 + Bx + 3$ and an eq | uation in A and/or B . | Al | [4] |
| State answer, e.g. $x = -1$ and no others B1 [2] 4 Separate variables and attempt integration of at least one side Obtain term $\ln(x+1)$ Obtain term $k \ln \sin 2\theta$, where $k = \pm 1, \pm 2$, or $\pm \frac{1}{2}$ M1 Obtain correct term $\frac{1}{2} \ln \sin 2\theta$ Evaluate a constant, or use limits $\theta = \frac{1}{12}\pi$, $x = 0$ in a solution containing terms $a \ln(x+1)$ and $b \ln \sin 2\theta$ M1 Obtain solution in any form, e.g. $\ln(x+1) = \frac{1}{2} \ln \sin 2\theta - \frac{1}{2} \ln \frac{1}{2}$ (f.t. on $k = \pm 1, \pm 2$, or $\pm \frac{1}{2}$) A1 $\sqrt{\frac{1}{2}}$ | | (ii) State a | nswer, e.g. $x = -3$ | | B1 | |
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| Obtain term $\ln(x+1)$ A1 Obtain term $k \ln \sin 2\theta$, where $k = \pm 1, \pm 2$, or $\pm \frac{1}{2}$ M1 Obtain correct term $\frac{1}{2} \ln \sin 2\theta$ A1 Evaluate a constant, or use limits $\theta = \frac{1}{12}\pi$, $x = 0$ in a solution containing terms $a \ln(x+1)$ and $b \ln \sin 2\theta$ M1 Obtain solution in any form, e.g. $\ln(x+1) = \frac{1}{2} \ln \sin 2\theta - \frac{1}{2} \ln \frac{1}{2}$ (f.t. on $k = \pm 1, \pm 2$, or $\pm \frac{1}{2}$) A1 $\sqrt{\frac{1}{2}}$ | 4 | Separate var | riables and attempt integration of at least one side | | M1 | |
| Obtain correct term $\frac{1}{2} \ln \sin 2\theta$ A1 Evaluate a constant, or use limits $\theta = \frac{1}{12} \pi$, $x = 0$ in a solution containing terms $a \ln(x + 1)$ and $b \ln \sin 2\theta$ M1 Obtain solution in any form, e.g. $\ln(x + 1) = \frac{1}{2} \ln \sin 2\theta - \frac{1}{2} \ln \frac{1}{2}$ (f.t. on $k = \pm 1, \pm 2$, or $\pm \frac{1}{2}$) A1 $\sqrt{\frac{1}{2}}$ | | Obtain term | ln(x+1) | | | |
| Evaluate a constant, or use limits $\theta = \frac{1}{12}\pi$, $x = 0$ in a solution containing terms $a \ln(x+1)$ and $b \ln \sin 2\theta$ M1 Obtain solution in any form, e.g. $\ln(x+1) = \frac{1}{2} \ln \sin 2\theta - \frac{1}{2} \ln \frac{1}{2}$ (f.t. on $k = \pm 1, \pm 2$, or $\pm \frac{1}{2}$) A1 $\sqrt{\frac{1}{2}}$ | | | | | | |
| b ln sin 2θ M1 Obtain solution in any form, e.g. $\ln(x+1) = \frac{1}{2} \ln \sin 2\theta - \frac{1}{2} \ln \frac{1}{2}$ (f.t. on $k = \pm 1, \pm 2$, or $\pm \frac{1}{2}$) A1 $\sqrt{\frac{1}{2}}$ | | | 2 | erms $a \ln(x+1)$ and | | |
| | | | | | M1 | |
| Rearrange and obtain $x = \sqrt{(2\sin 2\theta)} - 1$, or simple equivalent A1 [7] | | Obtain solu | tion in any form, e.g. $\ln(x+1) = \frac{1}{2} \ln \sin 2\theta - \frac{1}{2} \ln \frac{1}{2}$ (f.t. on $k = \frac{1}{2} \ln \frac{1}{2}$) | $\pm 1, \pm 2, \text{ or } \pm \frac{1}{2})$ | A 1√ | |
| | | Rearrange a | nd obtain $x = \sqrt{(2\sin 2\theta)} - 1$, or simple equivalent | | A1 | [7] |

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| 5 | (i) | | ognisable sketch of a relevant graph over the given interval e other relevant graph and justify the given statement | | B1 B1 | [2] |
| | (ii) | Consider | the sign of sec $x - (3 - \frac{1}{2} x^2)$ at $x = 1$ and $x = 1.4$, or equival | ent | M1 | |
| | | Complete | e the argument with correct calculated values | | A1 | [2] |
| | (iii) | Convert ti | he given equation to $\sec x = 3 - \frac{1}{2}x^2$ or work <i>vice versa</i> | | B1 | [1] |
| | (iv) | Obtain final answer 1.13 Show sufficient iterations to 4 d.p. to justify 1.13 to 2 d.p., or show there is a sign change | | M1 A1 | | |
| | | in the inte | erval (1.125, 1.135) cessive evaluation of the iterative function with $x = 1, 2,$ | | A1 | [3] |
| 6 | (i) | State or in | $mply R = \sqrt{10}$ | | B1 | |
| | | | Formulae to find α | | M1 | |
| | | | = 71.57° with no errors seen allow radians in this part. If the only trig error is a sign err | or in $\cos(x - \alpha)$ give | A1 | [3] |
| | (ii) | Evaluate | $\cos^{-1}(2/\sqrt{10})$ correctly to at least 1 d.p. (50.7684°) (All | ow 50.7° here) | В1√ | |
| | (11) | | an appropriate method to find a value of 2θ in $0^{\circ} < 2\theta < 18$ | · | M1 | |
| | | • | answer for θ in the given range, e.g. $\theta = 61.2^{\circ}$ | | A1 | |
| | | Use an ap | propriate method to find another value of 2θ in the above ra | inge | M1 | |
| | | [Ignore as [Treat and [SR: The | scond angle, e.g. $\theta = 10.4^{\circ}$, and no others in the given range inswers outside the given range.] swers in radians as a misread and deduct A1 from the answer use of correct trig formulae to obtain a 3-term quadrater. | tic in tan θ , $\sin^2 2\theta$, | | [5] |
| | | in the giv | tan 2θ earns M1; then A1 for a correct quadratic, M1 for or en range, and A1 + A1 for the two correct answers (candida spurious roots to get the final A1).] | | | |

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|---|-------|---|----------------------|-----|--|
| 7 | (i) | Use a correct method to express \overrightarrow{OP} in terms of λ Obtain the given answer | M1 A1 | [2] | |
| | (ii) | EITHER: Use correct method to express scalar product of \overrightarrow{OA} and \overrightarrow{OP} , or \overrightarrow{OB} and \overrightarrow{OP} in terms of λ Using the correct method for the moduli, divide scalar products by products of moduli and express $\cos AOP = \cos BOP$ in terms of λ , or in terms of λ and OP Use correct method to express $OA^2 + OP^2 - AP^2$, or $OB^2 + OP^2 - BP^2$ in terms of λ Using the correct method for the moduli, divide each expression by twice the product of the relevant moduli and express $\cos AOP = \cos BOP$ in terms of λ , or λ and OP | M1* | | |
| | | Obtain a correct equation in any form, e.g. $\frac{9+2\lambda}{3\sqrt{(9+4\lambda+12\lambda^2)}} = \frac{11+14\lambda}{5\sqrt{(9+4\lambda+12\lambda^2)}}$ | A1 | | |
| | | Solve for λ M1 Obtain $\lambda = \frac{3}{8}$ [SR: The M1* can also be earned by equating $\cos AOP$ or $\cos BOP$ to a sound attempt at $\cos \frac{1}{2} AOB$ and obtaining an equation in λ . The exact value of the cosine is $\sqrt{(13/15)}$, but accept non-exact working giving a value of λ which rounds to 0.375, provided the spurious negative root of the quadratic in λ is rejected.] | | [5] | |
| | | [SR: Allow a solution reaching $\lambda = \frac{3}{8}$ after cancelling identical incorrect expressions for <i>OP</i> to score 4/5. The marking will run M1M1A0M1A1, or M1M1A1M1A0 in succases.] | | | |
| | (iii) | Verify the given statement correctly | B1 | [1] | |
| 8 | (i) | Use any relevant method to determine a constant Obtain one of the values $A = 3$, $B = 4$, $C = 0$ Obtain a second value Obtain the third value | M1 A1 A1 A1 | [4] | |
| | (ii) | Integrate and obtain term $-3 \ln(2-x)$ Integrate and obtain term $k \ln(4+x^2)$ Obtain term $2 \ln(4+x^2)$ Substitute correct limits correctly in a complete integral of the form | B1√ M1 A1√ | | |
| | | Substitute correct limits correctly in a complete integral of the form $a \ln(2-x) + b \ln(4+x^2)$, $ab \neq 0$ Obtain given anywer following full and correct working | M1 | [5] | |

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A1

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Obtain given answer following full and correct working

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| 9 | (i) | Equate de | act rule arrect derivative in any form erivative to zero and solve for x swer $x = e^{-\frac{1}{2}}$, or equivalent | | M1 A1 M1 A1 | |
| | | Obtain an | swer $y = -\frac{1}{2} e^{-1}$, or equivalent | | A1 | [5] |
| | (ii) | Attempt i | ntegration by parts reaching $kx^3 \ln x \pm k \int x^3 \cdot \frac{1}{x} dx$ | | M1* | |
| | | Obtain $\frac{1}{3}$. | $x^3 \ln x - \frac{1}{3} \int x^2 dx$, or equivalent | | A1 | |
| | | Integrate | again and obtain $\frac{1}{3}x^3 \ln x - \frac{1}{9}x^3$, or equivalent | | A1 | |
| | | Use limits | s $x = 1$ and $x = e$, having integrated twice swer $\frac{1}{9}(2e^3 + 1)$, or exact equivalent | | M1(dep*) A1 | [5] |
| | | | attempt reaching $ax^2 (x \ln x - x) + b \int 2x(x \ln x - x) dx$ score | es M1. Then give | the | |
| | | | or $I = x^2 (x \ln x - x) - 2I + \int 2x^2 dx$, or equivalent.] | | | |
| | | | J | | | |
| 10 | (a) | EITHER: | Square $x + iy$ and equate real and imaginary parts to 1 and | $1 - 2\sqrt{6}$ respective | ely M1* | |
| | | | Obtain $x^2 - y^2 = 1$ and $2xy = -2\sqrt{6}$ | | A1 | |
| | | | Eliminate one variable and find an equation in the other Obtain $x^4 - x^2 - 6 = 0$ or $y^4 + y^2 - 6 = 0$, or 3-term equivalence of $\sqrt{2}$ | | M1(dep*) A1 | |
| | | 0.5 | Obtain answers $\pm (\sqrt{3} - i\sqrt{2})$ | \(\sigma\) \(\sigma\) \(\sigma\) | A1 | [5] |
| | | OR: | Denoting $1-2\sqrt{6i}$ by $R \operatorname{cis} \theta$, state, or imply, square ro | ots are $\pm \sqrt{R} \operatorname{cis}(\frac{1}{2})$ | | |
| | | | and find values of R and either $\cos \theta$ or $\sin \theta$ or $\tan \theta$ Obtain $\pm \sqrt{5} (\cos \frac{1}{2} \theta + i \sin \frac{1}{2} \theta)$, and $\cos \theta = \frac{1}{5}$ or | $\sin \theta = -\frac{2\sqrt{6}}{2}$ | M1* | |
| | | | $\tan \theta = -2\sqrt{6}$ | $\frac{1}{5}$ | A1 | |
| | | | Use correct method to find an exact value of $\cos \frac{1}{2}\theta$ or si | $n - \theta$ | M1(dep*) | |
| | | | Obtain $\cos \frac{1}{2}\theta = \pm \sqrt{\frac{3}{5}}$ and $\sin \frac{1}{2}\theta = \pm \sqrt{\frac{2}{5}}$, or equivalent | 2 | A1 | |
| | | | Obtain answers $\pm (\sqrt{3} - i\sqrt{2})$, or equivalent | | A1 | |
| | | | [Condone omission of \pm except in the final answers.] | | | |
| | <i>a</i> ` | CI. | | | D .1 | |
| | (b) | | nt representing 3i on a sketch of an Argand diagram arcle with centre at the point representing 3i and radius 2 | | B1 B1√ | |
| | | Shade the | interior of the circle | | B1√ | |
| | | - | a complete method for finding the greatest value of arg z swer 131.8° or 2.30 (or 2.3) radians | | M1 A1 | [5] |
| | | | s on solutions where the centre is at the point representing — | 2; 1 | AI | [5] |

[The f.t. is on solutions where the centre is at the point representing –3i.]