#### UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Subsidiary Level and GCE Advanced Level

# MARK SCHEME for the May/June 2009 question paper for the guidance of teachers

## 9709 MATHEMATICS

9709/03

Paper 3, maximum raw mark 75

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes must be read in conjunction with the question papers and the report on the examination.

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### **Mark Scheme Notes**

Marks are of the following three types:

- M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B Mark for a correct result or statement independent of method marks.
- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep\*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.
- Note: B2 or A2 means that the candidate can earn 2 or 0.
   B2/1/0 means that the candidate can earn anything from 0 to 2.

The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.
- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *q* equal to 9.8 or 9.81 instead of 10.

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The following abbreviations may be used in a mark scheme or used on the scripts:

AEF	Any Equivalent Form (of answer is equally acceptable)
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
BOD	Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)
CAO	Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)
CWO	Correct Working Only – often written by a 'fortuitous' answer
ISW	Ignore Subsequent Working
MR	Misread
PA	Premature Approximation (resulting in basically correct work that is insufficiently accurate)
sos	See Other Solution (the candidate makes a better attempt at the same question)
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

## **Penalties**

- MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through  $\sqrt{\phantom{0}}$ " marks. MR is not applied when the candidate misreads his own figures this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.
- PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

	Pag	je 4	Mark Scheme: Teachers' version	Syllabus	Paper	
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1	Carr		$y + e^{-x} = e^2$ thod for finding $\pm x$ from $e^{\pm x} = k$ , where $k > 0$ , following	ng sound ln	B1 M1	
	Obta Obta [The	ain x = -1 $ain answe$ $ain answer$	$ln(e^2 - 2)$ , or equivalent expression for $x$ er $x = -1.68$ must be given to 2 decimal places] is available for attempts starting with $2 + e^{-x} = 10^2$ ]		A1 A1	4
2	` ′		imply 3 of the 4 ordinates 1, 1.069389, 1.290994, 1		B1	
		Use corre	ect formula, or equivalent, with $h = \frac{1}{12}\pi$ and four ordin	ates	M1	
			nswer 0.98 with no errors seen $h = 0.26$ but not $h = 15$ when awarding the M1]		A1	3
		[SR: if o	nly $\sqrt{\frac{5}{3}}$ and/or $\sqrt{3}$ are given, and decimals are not seen	n, the B1 is avail	able]	
			ations with 2 or 4 intervals can score only the M1 for a			
	(ii)	Justify st	catement that the second estimate would be less than $E$		B1	1
3		Use a co	$A = 1/\tan A$ or $\cos A/\sin A$ and/or $\csc A = 1/\sin A$ on at largest double angle formula or the $\sin(A - B)$ formula at iven result		M1 M1 A1	3
	, ,	Obtain a	It $\theta = 2$ for $\theta$ and obtain answer 26.6° answer 206.6° and no others in the given range answers outside the given range. Treat answers given in	radians as a mis	B1 B1√ sread]	2
4		-	e signs of $x^3 - 2x - 2$ when $x = 1$ and $x = 2$ , or equivaler the argument with correct calculations	nt	M1 A1	2
	(ii)	State or i Rearrang	imply the equation $x = (2x^3 + 2) / (3x^2 - 2)$ ge this in the form $x^3 - 2x - 2 = 0$ , or work <i>vice versa</i>		B1 B1	2
		Obtain fi Show su	terative formula correctly at least once with $x_n > 0$ and answer 1.77 fficient iterations to 4 d.p. to justify its accuracy to 2 d. there is a sign change	p.,	M1 A1	
			terval (1.765, 1.775)		A1	3

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(i) State correct first two terms of the expansion of  $(1+ax)^{\frac{2}{3}}$ , i.e.  $1+\frac{2}{3}ax$ **B**1 5

Form an expression for the coefficient of x in the expansion of  $(1+2x)(1+ax)^{\frac{2}{3}}$ and equate it to zero

M1Obtain a = -3**A**1 3

(ii) Obtain correct unsimplified terms in  $x^2$  and  $x^3$  in the expansion of  $(1-3x)^{\frac{2}{3}}$ 

or  $(1 + ax)^{\frac{2}{3}}$  $B1\sqrt{+}B1\sqrt{}$ 

Carry out multiplication by 1 + 2x obtaining two terms in  $x^3$ M1

Obtain final answer  $-\frac{10}{3}x^3$ , or equivalent **A**1

[Symbolic binomial coefficients, e.g.  $\begin{pmatrix} \frac{2}{3} \\ 1 \end{pmatrix}$ , are not acceptable for the B marks in (i) or (ii)]

(i) EITHER State  $\frac{dx}{dt} = -3a\cos^2 t \sin t$  or  $\frac{dy}{dt} = 3a\sin^2 t \cos t$ , or equivalent **B**1

> Use  $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$ M1

State  $\frac{2}{3}x^{-\frac{1}{3}}dx$  or  $\frac{2}{3}y^{-\frac{1}{3}}dy$  as differentials of  $x^{\frac{2}{3}}$  or  $y^{\frac{2}{3}}$  respectively, **OR** or equivalent **B**1

Obtain  $\frac{dy}{dt}$  in terms of t, having taken the differential of a constant to be zero M1

- Obtain  $\frac{dy}{dx}$  in any correct form **A**1 3
- M1(ii) Form the equation of the tangent Obtain the equation in any correct form **A**1

Obtain the given answer **A**1 3

(iii) State the x-coordinate of X or the y-coordinate of Y in any correct form **B**1 Obtain the given answer with no errors seen

2

4

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7 (i) Use quadratic formula, or completing the square, or the substitution z = x + iy

to find a root, using  $i^2 = -1$ 

M1

Obtain a root, e.g.  $1 - \sqrt{3}i$ 

**A**1

Obtain the other root, e.g.  $-1 - \sqrt{3}i$ 

A1

3

1

3

1

(ii) Represent both roots on an Argand diagram in relatively correct positions

B1√

(iii) State modulus of both roots is 2

B1√

State argument of  $1 - \sqrt{3}i$  is  $-60^{\circ}$  (or  $300^{\circ}$ ,  $-\frac{1}{3}\pi$ ,  $-\frac{5}{3}\pi$ )

B1√

State argument of  $-1-\sqrt{3}i$  is  $-120^{\circ}$  (or  $240^{\circ}$ ,  $-\frac{2}{3}\pi$ ,  $-\frac{4}{3}\pi$ )

B1√

(iv) Give a complete justification of the statement

B1

[The A marks in (i) are for the final versions of the roots. Allow  $(\pm 2 - 2\sqrt{3}i)/2$  as final answer. The remaining marks are only available for roots such that  $xy \neq 0$ .] [Treat answers to (iii) in polar form as a misread]

8 (i) State or imply the form  $\frac{A}{r} + \frac{B}{r^2} + \frac{C}{10-r}$ 

B1

Use any relevant method to determine a constant

M1 A1

Obtain one of the values A = 1, B = 10, C = 1Obtain the remaining two values

A1

4

6

[The form  $\frac{Dx + E}{v^2} + \frac{C}{10 - x}$  is acceptable and leads to D = 1, E = 10, C = 1]

(ii) Separate variables and attempt integration of both sides

M1

Obtain terms  $\ln x$ , -10/x,  $-\ln (10 - x)$ , or equivalent

 $A1\sqrt{+}A1\sqrt{+}A1\sqrt{-}$ 

Evaluate a constant or use limits x = 1, t = 0 with a solution containing

3 of the terms  $k \ln x$ , l/x,  $m \ln (10 - x)$  and t, or equivalent

M1

Obtain any correct expression for t, e.g.  $t = \ln\left(\frac{9x}{10-x}\right) - \frac{10}{x} + 10$ 

A1

[A separation of the form  $\frac{a dx}{x^2 (10-x)} = b dt$  is essential for the M1. The f.t. is on A, B, C]

[If A or B (D or E) omitted from the form of fractions, give B0M1A0A0 in (i); M1A1 $\sqrt{\text{A1}\sqrt{\text{M1A0}}}$  in (ii)]

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9	(i) EITHE	<b>CR</b> Substitute coordinates of general point of <i>l</i> in equation	n of plane and		
	(i) Lilii	equate constant terms, obtaining an equation in $b$ and	-	M1*	
		Obtain a correct equation, e.g. $8 + 2b - c = 1$		A1	
		Equate the coefficient of $t$ to zero, obtaining an equation	ion in $b$ and $c$	M1*	
		Obtain a correct equation, e.g. $4 - b - 2c = 0$		A1	
	OR	Substitute $(4, 2, -1)$ in the plane equation		M1*	
		Obtain a correct equation in b and c, e.g. $2b - c = -7$		A1	
		<b>EITHER</b> Find a second point on <i>l</i> and obtain an equ		M1*	
		Obtain a correct equation in $b$ and $c$ , e.g. $b$		A1	
		OR Calculate scalar product of a direction vec		M1*	
		a vector normal for the plane and equate t Obtain a correct equation for b and c	0 2010	A1	
		Solve for $b$ or for $c$	M	1(dep*)	
		Obtain $b = -2$ and $c = 3$	111	A1	6
	(ii) EITHE	<b>ER</b> Find $PQ$ for a point $Q$ on $l$ with parameter $t$ , e.g. $4\mathbf{i}$ –	$5\mathbf{k} + t(2\mathbf{i} - \mathbf{j} - 2)$	<b>k</b> ) B1	
		Calculate scalar product of $\overrightarrow{PQ}$ and a direction vector	r for <i>l</i> and		
		equate to zero		M1	
		Solve and obtain $t = -2$		A1	
		Carry out a complete method for finding the length of	$\overrightarrow{PQ}$	M1	
		Obtain the given answer $\sqrt{5}$ correctly	_	A1	
	OR 1	Calling $(4, 2, -1) A$ , state $\overrightarrow{AP}$ (or $\overrightarrow{PA}$ ) in component	form eq /i _ 51	k B1	
	OKI		=	A Di	
		Calculate vector product of $\overrightarrow{AP}$ and a direction vecto e.g. $(4\mathbf{i} - 5\mathbf{k}) \times (2\mathbf{i} - \mathbf{j} - 2\mathbf{k})$	r ior <i>i</i> ,	M1	
		Obtain correct answer, e.g. $-5\mathbf{i} - 2\mathbf{j} - 4\mathbf{k}$		A1	
		Divide modulus of the product by that of the direction	n vector	M1	
		Obtain the given answer correctly	1 ( 00001	A1	
		·			
	OR 2	State $\overrightarrow{AP}$ (or $\overrightarrow{PA}$ ) in component form	<b>→</b>	B1	
		Use a scalar product to find the projection of $\overrightarrow{AP}$ (or	PA) on $l$	M1	
		Obtain correct answer in any form, e.g. $\frac{18}{\sqrt{9}}$		A1	
		Use Pythagoras to find the perpendicular		M1	
		Obtain the given answer correctly		<b>A</b> 1	
	OR 3	State $\overrightarrow{AP}$ (or $\overrightarrow{PA}$ ) in component form		B1	
		Use a scalar product to find the cosine of <i>PAQ</i>		M1	
		Obtain correct answer in any form, e.g. $\frac{18}{\sqrt{41} \cdot \sqrt{9}}$		A1	
		Use trig to find the perpendicular		M1	
		Obtain the given answer correctly		A1	
		South the Biven this wer correctly		111	

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	OR 4	State $\overrightarrow{AP}$ (or $\overrightarrow{PA}$ ) in component form Find a second point $B$ on $l$ and use the cosine rule in	_	B1	
		to find the cosine of $A$ , $B$ or $P$ , or use a vector produc	t to find the area		
		of <i>APB</i> Obtain correct answer in any form		M1 A1	
		Use trig or area formula to find the perpendicular		M1	
		Obtain the given answer correctly		A1	
	OR 5	Find $\overrightarrow{PQ}$ for a point Q on l with parameter t, e.g. $4\mathbf{i}$ -	$-5\mathbf{k} + t(2\mathbf{i} - \mathbf{j} - 2$	<b>k</b> ) B1	
		Use correct method to express $PQ^2$ (or $PQ$ ) in terms of	of t	M1	
		Obtain a correct expression in any form, e.g. $(4 + 2t)^2$	$(-t)^2 + (-t)^2 + (-5 - 2)^2$	$(2t)^2$ A1	
		Carry out a complete method for finding its minimum	1	M1	_
		Obtain the given answer correctly		A1	5
0 (i)	EITHER	Use product and chain rule		M1	
( )		Obtain correct derivative in any form		A1	
	OR	Square and differentiate LHS by chain rule and RHS	by product rule		
		or as powers		M1	
	.d.,	Obtain correct result in any form		A1	
	Set $\frac{dy}{dx}$ e	qual to zero and make reasonable attempt to solve for	$x \neq 0$	M1	
	Obtain a	nswer $x = \sqrt{\frac{2}{3}}$ , or exact equivalent, correctly		A1	4
(ii)	State or i	mply $dx = \cos \theta d\theta$ or $\frac{dx}{d\theta} = \cos \theta$		B1	
	Substitut	e for x and dx throughout the integral $\int y dx$		M1	
	Obtain th	ne given form correctly with no errors seen		A1	3
(iii)	Attempt where <i>al</i>	integration and reach indefinite integral of the form $a\theta = 0$	$\theta + b\sin 4\theta$ ,	M1*	
		definite integral $\frac{1}{8}\theta - \frac{1}{32}\sin 4\theta$ , or equivalent		A1	
		e limits correctly	N.	11(dep*)	
		xact answer $\frac{1}{16}\pi$	1	A1	4
		10		Λ1	7
	_	g to carry out the change of limits is needed for the A nd, can be earned retrospectively if it is seen in part (iii)			
	11 Offitie	a, can be carned renospectively if it is seen in part (m)	J		