UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Level

MATHEMATICS **9709/03**

Paper 3 Pure Mathematics 3 **(P3)** **October/November 2007**

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

6606955897

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

This document consists of **3** printed pages and **1** blank page.

1 Find the exact value of the constant k for which $\int_1^k \frac{1}{2x-1}\,\mathrm{d}x = 1$. [4]

2 The polynomial $x^4 + 3x^2 + a$, where a is a constant, is denoted by $\mathrm{p}(x)$. It is given that $x^2 + x + 2$ is a factor of $\mathrm{p}(x)$. Find the value of a and the other quadratic factor of $\mathrm{p}(x)$. [4]

3 Use integration by parts to show that

$$\int_2^4 \ln x\,\mathrm{d}x = 6\ln 2 - 2.$$ [4]

4 The curve with equation $y = \mathrm{e}^{-x}\sin x$ has one stationary point for which $0 \leqslant x \leqslant \pi$.

(i) Find the x-coordinate of this point. [4]

(ii) Determine whether this point is a maximum or a minimum point. [2]

5 **(i)** Show that the equation

$$\tan(45^\circ + x) - \tan x = 2$$

can be written in the form

$$\tan^2 x + 2\tan x - 1 = 0.$$ [3]

(ii) Hence solve the equation

$$\tan(45^\circ + x) - \tan x = 2,$$

giving all solutions in the interval $0^\circ \leqslant x \leqslant 180^\circ$. [4]

6 **(i)** By sketching a suitable pair of graphs, show that the equation

$$2 - x = \ln x$$

has only one root. [2]

(ii) Verify by calculation that this root lies between 1.4 and 1.7. [2]

(iii) Show that this root also satisfies the equation

$$x = \tfrac{1}{3}(4 + x - 2\ln x).$$ [1]

(iv) Use the iterative formula

$$x_{n+1} = \tfrac{1}{3}(4 + x_n - 2\ln x_n),$$

with initial value $x_1 = 1.5$, to determine this root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

7 The number of insects in a population t days after the start of observations is denoted by N. The variation in the number of insects is modelled by a differential equation of the form

$$\frac{\mathrm{d}N}{\mathrm{d}t} = kN\cos(0.02t),$$

where k is a constant and N is taken to be a continuous variable. It is given that $N = 125$ when $t = 0$.

(i) Solve the differential equation, obtaining a relation between N, k and t. [5]

(ii) Given also that $N = 166$ when $t = 30$, find the value of k. [2]

(iii) Obtain an expression for N in terms of t, and find the least value of N predicted by this model. [3]

8 **(a)** The complex number z is given by $z = \dfrac{4 - 3\mathrm{i}}{1 - 2\mathrm{i}}$.

(i) Express z in the form $x + \mathrm{i}y$, where x and y are real. [2]

(ii) Find the modulus and argument of z. [2]

(b) Find the two square roots of the complex number $5 - 12\mathrm{i}$, giving your answers in the form $x + \mathrm{i}y$, where x and y are real. [6]

9 **(i)** Express $\dfrac{2 - x + 8x^2}{(1 - x)(1 + 2x)(2 + x)}$ in partial fractions. [5]

(ii) Hence obtain the expansion of $\dfrac{2 - x + 8x^2}{(1 - x)(1 + 2x)(2 + x)}$ in ascending powers of x, up to and including the term in x^2. [5]

10 The straight line l has equation $\mathbf{r} = \mathbf{i} + 6\mathbf{j} - 3\mathbf{k} + s(\mathbf{i} - 2\mathbf{j} + 2\mathbf{k})$. The plane p has equation $(\mathbf{r} - 3\mathbf{i}).(2\mathbf{i} - 3\mathbf{j} + 6\mathbf{k}) = 0$. The line l intersects the plane p at the point A.

(i) Find the position vector of A. [3]

(ii) Find the acute angle between l and p. [4]

(iii) Find a vector equation for the line which lies in p, passes through A and is perpendicular to l. [5]

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