UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education
Advanced Subsidiary Level and Advanced Level

HIGHER MATHEMATICS **8719/03**

MATHEMATICS **9709/03**

Paper 3 Pure Mathematics 3 **(P3)**

October/November 2005

1 hour 45 minutes

Additional materials: Answer Booklet/Paper
Graph paper
List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

This document consists of **3** printed pages and **1** blank page.

UNIVERSITY of CAMBRIDGE International Examinations

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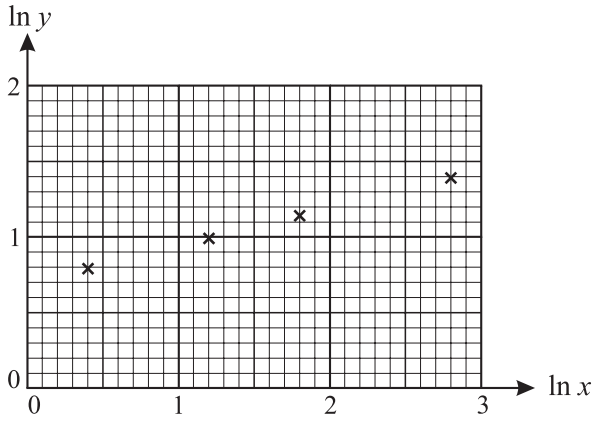
1 Given that a is a positive constant, solve the inequality

$$|x - 3a| > |x - a|.$$ [4]

2



Two variable quantities x and y are related by the equation $y = Ax^n$, where A and n are constants. The diagram shows the result of plotting $\ln y$ against $\ln x$ for four pairs of values of x and y. Use the diagram to estimate the values of A and n. [5]

3 The equation of a curve is $y = x + \cos 2x$. Find the x-coordinates of the stationary points of the curve for which $0 \leqslant x \leqslant \pi$, and determine the nature of each of these stationary points. [7]

4 The equation $x^3 - x - 3 = 0$ has one real root, α.

(i) Show that α lies between 1 and 2. [2]

Two iterative formulae derived from this equation are as follows:

$$x_{n+1} = x_n^3 - 3, \qquad (A)$$

$$x_{n+1} = (x_n + 3)^{\frac{1}{3}}. \qquad (B)$$

Each formula is used with initial value $x_1 = 1.5$.

(ii) Show that one of these formulae produces a sequence which fails to converge, and use the other formula to calculate α correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [5]

5 By expressing $8 \sin \theta - 6 \cos \theta$ in the form $R \sin(\theta - \alpha)$, solve the equation

$$8 \sin \theta - 6 \cos \theta = 7,$$

for $0^\circ \leqslant \theta \leqslant 360^\circ$. [7]

6 **(i)** Use the substitution $x = \sin^2\theta$ to show that

$$\int \sqrt{\left(\frac{x}{1-x}\right)}\,\mathrm{d}x = \int 2\sin^2\theta\,\mathrm{d}\theta. \qquad [4]$$

(ii) Hence find the exact value of

$$\int_0^{\frac{1}{4}} \sqrt{\left(\frac{x}{1-x}\right)}\,\mathrm{d}x. \qquad [4]$$

7 The equation $2x^3 + x^2 + 25 = 0$ has one real root and two complex roots.

(i) Verify that $1 + 2\mathrm{i}$ is one of the complex roots. [3]

(ii) Write down the other complex root of the equation. [1]

(iii) Sketch an Argand diagram showing the point representing the complex number $1 + 2\mathrm{i}$. Show on the same diagram the set of points representing the complex numbers z which satisfy

$$|z| = |z - 1 - 2\mathrm{i}|. \qquad [4]$$

8 In a certain chemical reaction the amount, x grams, of a substance present is decreasing. The rate of decrease of x is proportional to the product of x and the time, t seconds, since the start of the reaction. Thus x and t satisfy the differential equation

$$\frac{\mathrm{d}x}{\mathrm{d}t} = -kxt,$$

where k is a positive constant. At the start of the reaction, when $t = 0$, $x = 100$.

(i) Solve this differential equation, obtaining a relation between x, k and t. [5]

(ii) 20 seconds after the start of the reaction the amount of substance present is 90 grams. Find the time after the start of the reaction at which the amount of substance present is 50 grams. [3]

9 **(i)** Express $\dfrac{3x^2 + x}{(x+2)(x^2+1)}$ in partial fractions. [5]

(ii) Hence obtain the expansion of $\dfrac{3x^2 + x}{(x+2)(x^2+1)}$ in ascending powers of x, up to and including the term in x^3. [5]

10 The straight line l passes through the points A and B with position vectors

$$2\mathbf{i} + 2\mathbf{j} + \mathbf{k} \qquad \text{and} \qquad \mathbf{i} + 4\mathbf{j} + 2\mathbf{k}$$

respectively. This line intersects the plane p with equation $x - 2y + 2z = 6$ at the point C.

(i) Find the position vector of C. [4]

(ii) Find the acute angle between l and p. [4]

(iii) Show that the perpendicular distance from A to p is equal to 2. [3]

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