Rewarding Learning

ADVANCED
General Certificate of Education
2015

Mathematics

Assessment Unit M4

assessing

Module M4: Mechanics 4

[AMM41]

WEDNESDAY 24 JUNE, MORNING

MARK
SCHEME

GCE ADVANCED/ADVANCED SUBSIDIARY (AS) MATHEMATICS

Introduction

The mark scheme normally provides the most popular solution to each question. Other solutions given by candidates are evaluated and credit given as appropriate; these alternative methods are not usually illustrated in the published mark scheme.

The marks awarded for each question are shown in the right-hand column and they are prefixed by the letters **M**, **W** and **MW** as appropriate. The key to the mark scheme is given below:

M indicates marks for correct method.

W indicates marks for working.

MW indicates marks for combined method and working.

The solution to a question gains marks for correct method and marks for an accurate working based on this method. Where the method is not correct no marks can be given.

A later part of a question may require a candidate to use an answer obtained from an earlier part of the same question. A candidate who gets the wrong answer to the earlier part and goes on to the later part is naturally unaware that the wrong data is being used and is actually undertaking the solution of a parallel problem from the point at which the error occurred. If such a candidate continues to apply correct method, then the candidate's individual working must be followed through from the error. If no further errors are made, then the candidate is penalised only for the initial error. Solutions containing two or more working or transcription errors are treated in the same way. This process is usually referred to as "follow-through marking" and allows a candidate to gain credit for that part of a solution which follows a working or transcription error.

Positive marking:

It is our intention to reward candidates for any demonstration of relevant knowledge, skills or understanding. For this reason we adopt a policy of **following through** their answers, that is, having penalised a candidate for an error, we mark the succeeding parts of the question using the candidate's value or answers and award marks accordingly.

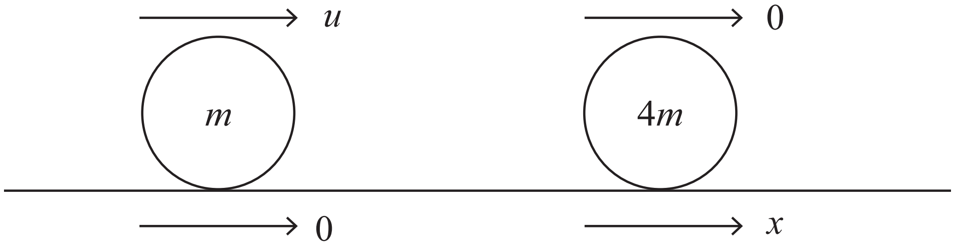
Some common examples of this occur in the following cases:

(a) a numerical error in one entry in a table of values might lead to several answers being incorrect, but these might not be essentially separate errors;

(b) readings taken from candidates' inaccurate graphs may not agree with the answers expected but might be consistent with the graphs drawn.

When the candidate misreads a question in such a way as to make the question easier only a proportion of the marks will be available (based on the professional judgement of the examining team).

1 **(i)** Collision between A and B:



Momentum is conserved: $mu + 0 = 0 + 4mx$ M1

$$u = 4x$$ MW1

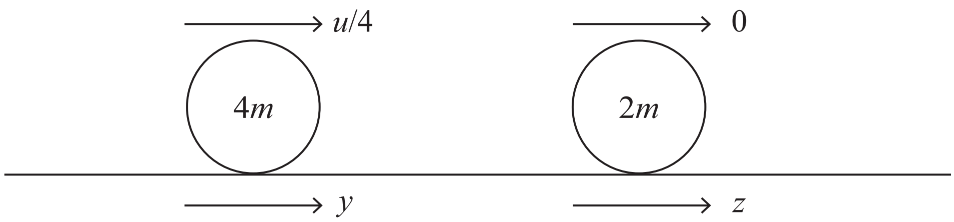
Newton's experimental law: $x - 0 = -e_{AB}(0 - u)$, M1 W1

where e_{AB} is the coefficient of restitution between A and B.

$$x = e_{AB}u$$

Hence, $\frac{u}{4} = e_{AB}u \Rightarrow e_{AB} = \frac{1}{4}$ MW1

(ii) Collision between B and C:



Momentum is conserved: $mu = 4my + 2mz$

$u = 4y + 2z$ MW1

Newton's experimental law: $z - y = -\frac{4}{5}\left(0 - \frac{u}{4}\right) = \frac{u}{5}$ MW1

$z = \frac{3u}{10}, y = \frac{u}{10}$ MW1 MW1

(iii) Total loss of kinetic energy:

$\frac{1}{2}mu^2 - \left(\frac{1}{2}\ 4my^2 + \frac{1}{2}\ 2mz^2\right)$ M1

$= \frac{m}{2}(u^2 - 4y^2 - 2z^2)$

$= \frac{m}{2}u^2\left(1 - \frac{1}{25} - \frac{9}{50}\right)$ MW1

$= \frac{39m}{100}u^2$ W1

AVAILABLE MARKS

12

		AVAILABLE MARKS

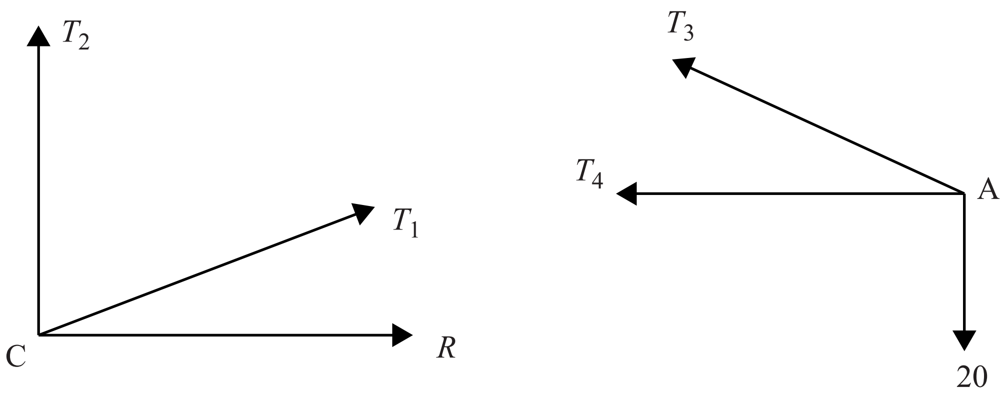
2 **(i)** Given the contact at C is smooth, the reaction force on the pin at C must be perpendicular to the surface of the wall, i.e. it must be a horizontal force. M1

(ii) Let R N be the reaction on the pin at C.

M(B): $R(0.5) = 20(0.5 + 0.5 \cos 30°)$ M1 W1

$R = 37.3$ (3 s.f.) W1

(iii)



T_1N, T_2N, are the forces in the rods meeting at C.

At C, R (→): $R + T_1 \cos 30° = 0$

$T_1 = -43.1$ (3 s.f.) M1 W1

R(↑): $T_2 + T_1 \cos 60° = 0$

$T_2 = 21.5$ (3 s.f.) MW1

T_3N, T_4N are the forces in the rods meeting at A.

At A, R(↑): $T_3 \cos 75° - 20 = 0$

$T_3 = 77.3$ (3 s.f.) MW1

R(→): $T_4 + T_3 \cos 15° = 0$

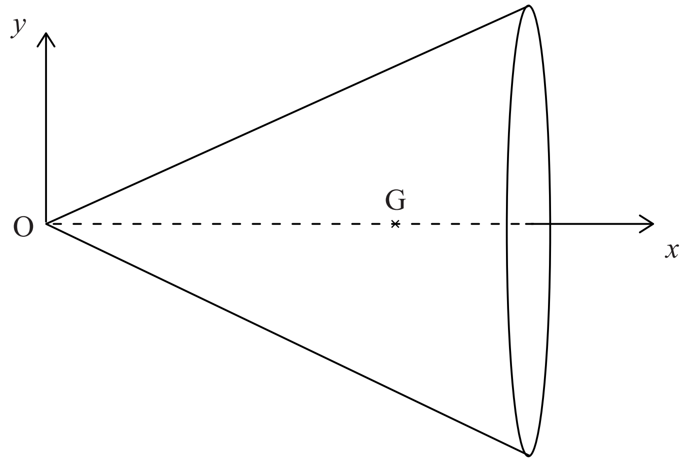
$T_4 = -74.6$ (3 s.f.) MW1

(iv) T_2N, T_3N represent tensions, so the rods CB and AB could be replaced by light ropes. M1 W1 11

		AVAILABLE MARKS
3 **(i)** Gravitational force $= \dfrac{GM_SM_M}{(R+r)^2}$	MW1	
(ii) $F_{EM} = \dfrac{GM_EM_M}{r^2}$		
$F_{SM} = \dfrac{GM_SM_M}{(R+r)^2}$		
$\dfrac{F_{EM}}{F_{SM}} = \dfrac{M_E(R+r)^2}{M_S\, r^2} = \dfrac{M_E}{M_S}\left(\dfrac{R}{r} + 1\right)^2$	M1 W1	
(iii) $\dfrac{F_{EM}}{F_{SM}} = \dfrac{1}{3.328 \times 10^5}(3.896 \times 10^2 + 1)^2 = 0.458$ (3 s.f.)	M1 W1	
(iv) $F_{SM} = \dfrac{GM_SM_M}{(\mathrm{R}-r)^2}$	M1	
$\dfrac{F_{EM}}{F_{SM}} = \dfrac{M_E}{M_S}\left(\dfrac{R}{r} - 1\right)^2$	MW1	
$\dfrac{F_{EM}}{F_{SM}} = \dfrac{1}{3.328 \times 10^5}(3.896 \times 10^2 - 1)^2 = 0.454$ (3 s.f.)	W1	8
4 **(i)** **M**(O): $5(2a) + 2(3) - 3(4a) - 4(2) + 6(2)$	M1 W2	
$= 10 - 2a$	MW1	
(ii) $10 - 2a = 0 \Rightarrow a = 5$	MW1	
(iii) $\lvert 10 - 2a \rvert = 16$	M1	
$10 - 2a = 16 \Rightarrow a = -3$	MW1	
$2a - 10 = 16 \Rightarrow a = 13$	MW1	8

	AVAILABLE MARKS

5 (i)



The centre of mass lies along Ox.

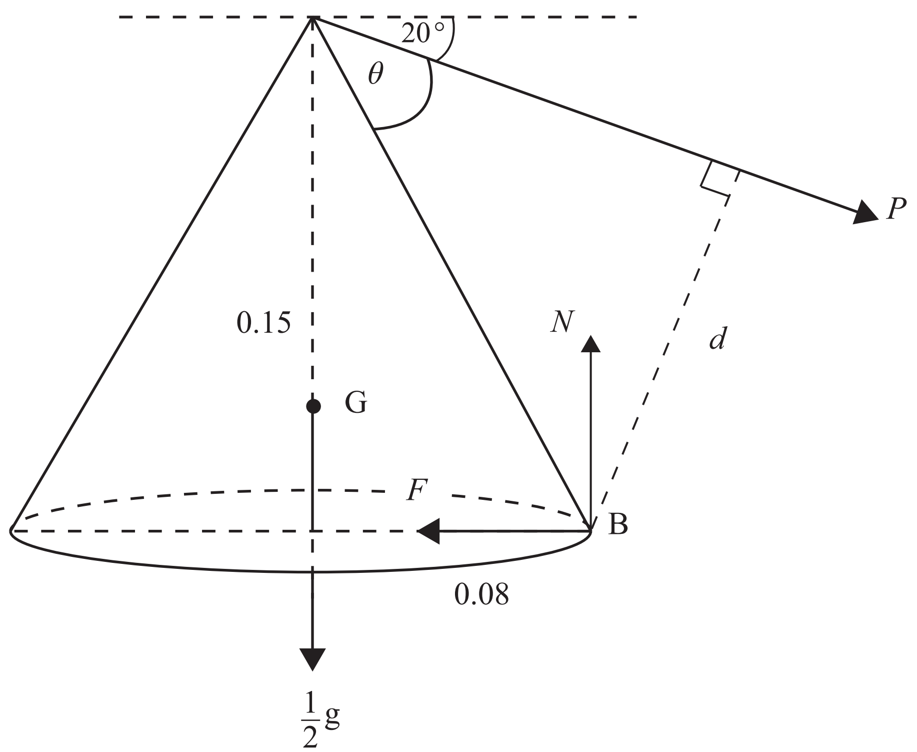
M(Oy): $\frac{1}{3}\pi r^2 h\bar{x} = \int_0^h \pi y^2 x \, dx$ M2 W2

$\frac{1}{3}\pi r^2 h\bar{x} = \int_0^h \pi \left(\frac{rx}{h}\right)^2 x \, dx$

$\frac{1}{3}\pi r^2 h\bar{x} = \pi \left(\frac{r}{h}\right)^2 \left[\frac{x^4}{4}\right]_0^h$ MW1

$\bar{x} = \frac{3h}{4} \Rightarrow$ G centre of mass is $\frac{h}{4}$ from base. W1

(ii)



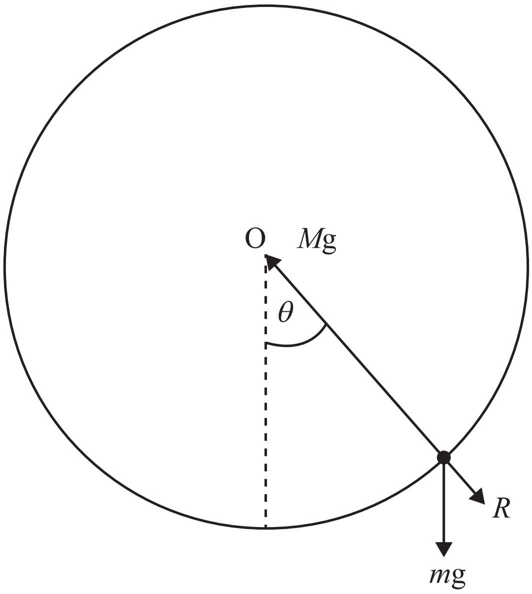
M(B): $\frac{1}{2}g(0.08) = Pd$ M1 W1

$d = (0.17 \sin\theta) = 0.17 \sin\left(90° - 20° - \tan^{-1}\left(\frac{8}{15}\right)\right)$ MW1

$P = \frac{\frac{1}{2}g(0.08)}{0.11359\ldots} = 3.45$ (3 s.f.) W1 10

				AVAILABLE MARKS
6	(i)	$[v] = [LT^{-1}]$, $[g] = [LT^{-2}]$, $[\lambda] = [L]$ and $[\rho] = [ML^{-3}]$	MW2	
		$[LT^{-1}] = [L]^a [LT^{-2}]^b [ML^{-3}]^c [MT^{-2}]^d$	M1	
		$[LT^{-1}] = [M^{c+d}] [L^{a+b-3c}] [T^{-2b-2d}]$	W1	
		Equating powers of M gives $c + d = 0$	MW1	
	(ii)	Equating powers of L and T gives	M1	
		$a + b - 3c = 1$ and		
		$-2b - 2d = -1 \Rightarrow b = \frac{1}{2} - d$	W1	
		Hence, $a = 1 - b + 3c = 1 - \frac{1}{2} + d - 3d$		
		$\Rightarrow a = \frac{1}{2} - 2d$	W1	
	(iii)	$v = k\,\lambda^{\frac{1}{2}-2d} g^{\frac{1}{2}-d} \rho^{-d} S^d$	M1	
		$v = k\sqrt{\lambda g}\left(\frac{S}{\rho\lambda^2 g}\right)^d$	W1	
	(iv)	If v is independent of S, then $d = 0$		
		$v = k\sqrt{\lambda g}$	MW1	
	(v)	If v is independent of g, then $d = \frac{1}{2}$	M1	
		$v = k\sqrt{\frac{S}{\rho\lambda}}$	MW1	13

		AVAILABLE MARKS
7 **(i)** Conservation of mechanical energy:		
$(K.E.)_A + (G.P.E.)_A = (K.E.)_P + (G.P.E)_P$		
$\frac{1}{2}mu^2 + 0 = \frac{1}{2}mv^2 + mgr(1 - \cos\theta)$	MW1	
$v^2 = u^2 - 2gr(1 - \cos\theta)$	W1	
$u^2 = 8gr \Rightarrow v^2 = 6gr + 2gr(\cos\theta)$	MW1	
When $\theta = 180°$, $6gr - 2gr > 0$		
So, $v > 0$ and the bead makes complete revolutions on the wire.	MW1	
(ii)		



Resultant force towards O (taking R to act radically outwards):		
$Mg - R - mg\cos\theta = m\frac{v^2}{r}$	M1 M1 W1	
$R = Mg - mg\cos\theta - m\frac{v^2}{r}$		
$R = Mg - mg\cos\theta - m(6g + 2g\cos\theta)$	MW1	
$R = Mg - 3mg\cos\theta - 6mg$		
Since R can also act radically inwards, $R = \pm(Mg - 3mg\cos\theta - 6mg)$	MW1	
(iii) $R = 0 \quad \Rightarrow \cos\theta = \frac{M - 6m}{3m}$	M1 W1	
$\Rightarrow -1 \leqslant \frac{M - 6m}{3m} \leqslant 1$	M1	
$\Rightarrow -3m \leqslant M - 6m \leqslant 3m$		
$\Rightarrow 3m \leqslant M \leqslant 9m$	W1	13
	Total	**75**