Rewarding Learning

ADVANCED
General Certificate of Education
2014

Mathematics

Assessment Unit M2

assessing

Module M2: Mechanics 2

[AMM21]

AMM21

THURSDAY 12 JUNE, AFTERNOON

TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.
Answer **all seven** questions.
Show clearly the full development of your answers.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or a scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
Answers should include diagrams where appropriate and marks may be awarded for them.
Take $g = 9.8\,\text{m}\,\text{s}^{-2}$, unless specified otherwise.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

Answer all seven questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

1 Two forces $\mathbf{F}_1$ and $\mathbf{F}_2$ act on a particle of mass 6 kg, where $\mathbf{F}_1 = (2\mathbf{i} - 8\mathbf{j} - \mathbf{k})$ N.
The resultant of these two forces gives the particle an acceleration of $(2\mathbf{i} - 5\mathbf{j} + 2\mathbf{k})$ m s^{-2}

(i) Find $\mathbf{F}_2$ [4]

A third force $\mathbf{F}_3$ now acts on the particle together with $\mathbf{F}_1$ and $\mathbf{F}_2$
The resultant of these three forces causes the particle to move with a constant velocity.

(ii) Find $\mathbf{F}_3$ [2]

2 **Fig. 1** below shows a projectile fired with an initial velocity of u m s^{-1} at an angle θ above the horizontal.
The horizontal range of the projectile is R metres.

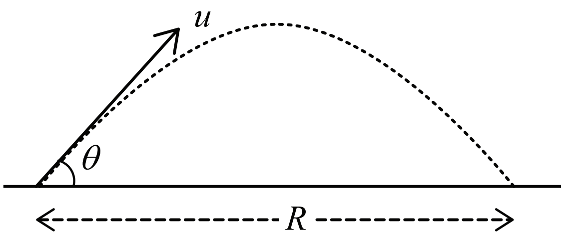

Fig. 1

(i) Show that the total time of flight, T seconds, of the projectile is given by

$$T = \frac{2u \sin \theta}{g}$$

[3]

(ii) Hence, show that

$$R = \frac{u^2 \sin 2\theta}{g}$$

[4]

(iii) For $u = 20$ m s^{-1}, find the **maximum** value of R. [2]

(iv) State one modelling assumption you have made in answering this question. [1]

3 A stunt rider, Rodney, is travelling on a smooth bend in a track which is in the form of an arc of a circle of radius 200 m.
The combined mass of Rodney and his motorcycle is 180 kg.
The bend in the track is banked at an angle of 35° to the horizontal, as shown in **Fig. 2** below.

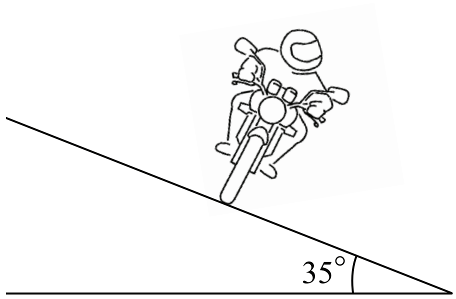

Fig. 2

Model Rodney and his motorcycle as a single particle.
(You may assume there are no sideways frictional forces between the tyres of the motorcycle and the track.)

(i) Draw a diagram showing all the external forces acting on this particle. [2]

While negotiating the bend, he travels at a constant speed of v m s^{-1}

(ii) Find the normal reaction between the road and the tyres of the motorcycle. [3]

(iii) Find v. [5]

4 A car of mass 1400 kg is travelling on a straight horizontal road.
The engine of the car is working at a constant rate of 80 kW and the resistive forces opposing the motion of the car are constant and total 1600 N.

(i) Find the acceleration of the car when it is travelling at 25 m s^{-1} [4]

(ii) Find, in kJ, the kinetic energy of the car when it is travelling at maximum speed. [6]

The car now ascends a hill which is straight and inclined at θ° to the horizontal.
The power output and resistive forces remain the same as before.
When the car is travelling at 20 m s^{-1} up the hill, it has a retardation of 0.3 m s^{-2}

(iii) Find θ. [5]

5 **[In this question, take g to be 10 $m\,s^{-2}$]**

An emergency parcel is to be dropped from a hovering helicopter to stranded hillwalkers. The parcel of mass m kg is dropped from rest and falls vertically under gravity.

The parcel is subject to air resistance of magnitude $\frac{mv^2}{60}$ newtons, where v m s^{-1} is the speed of the parcel, when it has dropped x metres.

(i) Show that the equation of motion of the parcel can be modelled by the differential equation

$$60v\frac{dv}{dx} = 600 - v^2$$

[4]

If the parcel is to land intact, its speed of impact with the ground below must be no more than 14 $m\,s^{-1}$

(ii) Find the maximum height from which the parcel can be dropped if it is to land intact. [8]

6 A particle, P, is moving in the **i**–**j** plane so that its velocity, **v** $m\,s^{-1}$, at any time t seconds is given by

$$\mathbf{v} = (4\cos t)\mathbf{i} + (3 + 2\sin t)\mathbf{j}$$

(i) Find the acceleration of P when $t = \pi$ [5]

When $t = 0$, the displacement of P from the origin O is $(2\mathbf{i} - 3\mathbf{j})$ m.

(ii) Find an expression for **s**, the displacement of the particle from O, at any time t. [5]

(iii) Find t when the particle first crosses the **j**-axis. [2]

7 **Fig. 3** below shows two balls A and B of masses m kg and $3m$ kg respectively connected by a light inextensible string which passes over a smooth fixed pulley.
Ball A is on a smooth plane inclined at 30° to the horizontal, and ball B hangs freely.

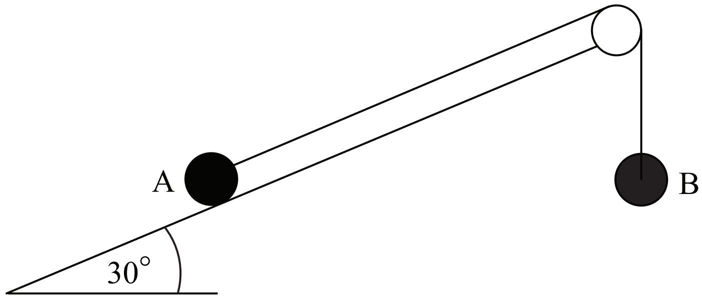

Fig. 3

Initially, A and B are held at the **same** horizontal level with the string taut.
They are released from rest.

(i) By using the Law of Conservation of Mechanical Energy, find in terms of g and l, the velocity v m s^{-1} of B when it has fallen l metres.
(Assume A does not reach the pulley.) [8]

(ii) Explain why the work done by the normal reaction between the plane and A is 0 J. [1]

(iii) Explain why the work done by the tension in the string does not have to be considered. [1]

THIS IS THE END OF THE QUESTION PAPER