

ADVANCED General Certificate of Education 2014

# Mathematics

Assessment Unit F2 assessing Module FP2: Further Pure Mathematics 2 AMF21

## [AMF21]

## THURSDAY 12 JUNE, AFTERNOON

### TIME

1 hour 30 minutes.

## **INSTRUCTIONS TO CANDIDATES**

Write your Centre Number and Candidate Number on the Answer Booklet provided. Answer **all eight** questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

You are permitted to use a graphic or scientific calculator in this paper.

## INFORMATION FOR CANDIDATES

The total mark for this paper is 75

Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.

A copy of the Mathematical Formulae and Tables booklet is provided.

Throughout the paper the logarithmic notation used is  $\ln z$  where it is noted that  $\ln z \equiv \log_{e} z$ 

#### Answer all eight questions.

#### Show clearly the full development of your answers.

#### Answers should be given to three significant figures unless otherwise stated.

1 Given that

$$f(x) = \ln\left(1 + \sin x\right)$$

find the Maclaurin expansion for f(x) up to and including the term in  $x^2$  [6]

2 Find, in radians, the general solution of the equation

$$\sin 3\theta = \sin \theta \tag{6}$$

**3** A geometric series is given by

$$e^{3x} + 3e^{x} + 9e^{-x} + \dots \qquad x > 0$$

- (i) Find an expression for the *n*th term of the series. [3]
- (ii) Find the range of values of x for which the series has a sum to infinity. [4]
- (iii) Find the sum to infinity when  $x = \ln 2$  [4]

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4 (i) Given that

xy + 3y - x = 0

show that

$$(x+3)\,\frac{dy}{dx} + y = 1$$
[2]

(ii) Hence, using mathematical induction, prove that for  $n \ge 2$ 

$$(x+3)\frac{d^{n}y}{dx^{n}} + n\frac{d^{n-1}y}{dx^{n-1}} = 0$$
[5]

5 (i) Express in partial fractions

$$f(x) = \frac{2}{(x-1)^2 (x^2+1)}$$
[6]

(ii) Hence find the exact value of

$$\int_{2}^{3} \frac{2}{(x-1)^{2}(x^{2}+1)} \,\mathrm{d}x$$
 [5]

6 (i) Show that

$$(Z^n - e^{i\theta})(Z^n - e^{-i\theta}) \equiv Z^{2n} - 2Z^n \cos \theta + 1$$
  
where Z is a complex number. [2]

(ii) Hence or otherwise find in  $(\cos \theta + i \sin \theta)$  form the roots of the equation

$$Z^4 - Z^2 \sqrt{2} + 1 = 0$$
 [5]

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## [Turn over

7 (i) Show that an equation of the normal to the parabola  $y^2 = 4ax$  at the point  $P(at^2, 2at)$  is

$$y + tx = 2at + at^3 \tag{4}$$

The normal at P meets the curve again at the point  $Q(as^2, 2as)$ .

(ii) Show that if  $t \neq 0$ 

$$s = -\left(t + \frac{2}{t}\right) \tag{3}$$

(iii) Using (ii) show that

$$PQ^{2} = \frac{16a^{2}(t^{2}+1)^{3}}{t^{4}}$$
[4]

8 A particle P moves in a straight line so that its displacement *x* centimetres from a fixed point at time *t* seconds is given by the differential equation

$$\frac{\mathrm{d}^2 x}{\mathrm{d}t^2} + 4 \frac{\mathrm{d}x}{\mathrm{d}t} + 4x = \lambda \sin 2t$$

where  $\lambda$  is a constant.

(i) Given that 
$$x = 0$$
 and  $\frac{dx}{dt} = \frac{\lambda}{4}$  when  $t = 0$ , find x in terms of t and  $\lambda$ . [14]

(ii) As *t* becomes large describe what happens to the solution found in part (i). [2]

## THIS IS THE END OF THE QUESTION PAPER