



Rewarding Learning

ADVANCED
General Certificate of Education
2014

Mathematics

Assessment Unit C3

assessing

Module C3: Core Mathematics 3

[AMC31]

THURSDAY 15 MAY, AFTERNOON



TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.

Answer **all eight** questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75

Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.

A copy of the **Mathematical Formulae and Tables booklet** is provided.

Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

Answer all eight questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

- 1 Farmer Fred's field floods. The area under water adjoins a dyke. Fred takes measurements, in metres, 3 m apart, from the water's edge to the opposite boundary as shown in Fig. 1 below:

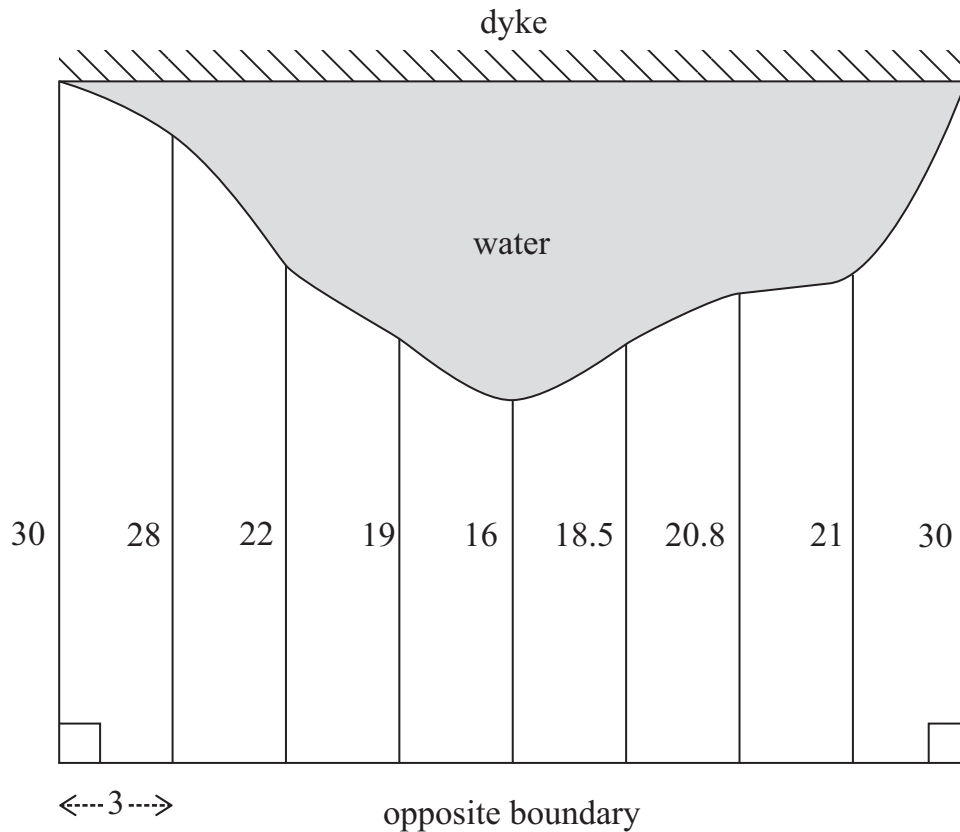


Fig. 1

Use Simpson's Rule to find an approximation to the surface area of the field which is not under water.

[4]

- 2 A curve is defined parametrically by

$$x = at \quad y = \frac{a}{1+t^2}$$

Derive a Cartesian equation of this curve.

[3]

3 Given the functions:

$$f(x) = \frac{2x^2 - x - 15}{2x - 5}$$

and

$$g(x) = \frac{x - 3}{4x^2 - 25}$$

write $\frac{f(x)}{g(x)}$ in the form $(ax + b)^2$, where a and b are integers. [5]

4 (a) (i) Express

$$\frac{2}{x^2 - 1}$$

in partial fractions. [5]

(ii) Hence, or otherwise, show that

$$\frac{1}{\sec \theta - 1} - \frac{1}{\sec \theta + 1} \equiv 2 \cot^2 \theta$$
 [4]

(b) (i) Expand

$$(1 + 5x)^{\frac{2}{5}}$$

in a binomial series up to and including the term in x^3 [4]

(ii) Hence, **write down** a similar expansion for

$$(1 - 5x)^{\frac{2}{5}}$$
 [2]

(iii) Hence, assuming x^4 and higher powers of x can be ignored, solve the equation

$$\sqrt[5]{(1 + 5x)^2} + \sqrt[5]{(1 - 5x)^2} = 1\frac{7}{8}$$
 [3]

5 (a) Find

$$\int \left(\operatorname{cosec}^2 2x + \sqrt{x} - \frac{3}{x} + e^{-3x} \right) dx \quad [5]$$

(b) The blade of a knife can be modelled as the area bounded by the curves $y = 3 \cos x$ and $y = \frac{\pi^2}{4} - x^2$ and the y -axis, as shown shaded in **Fig. 2** below.

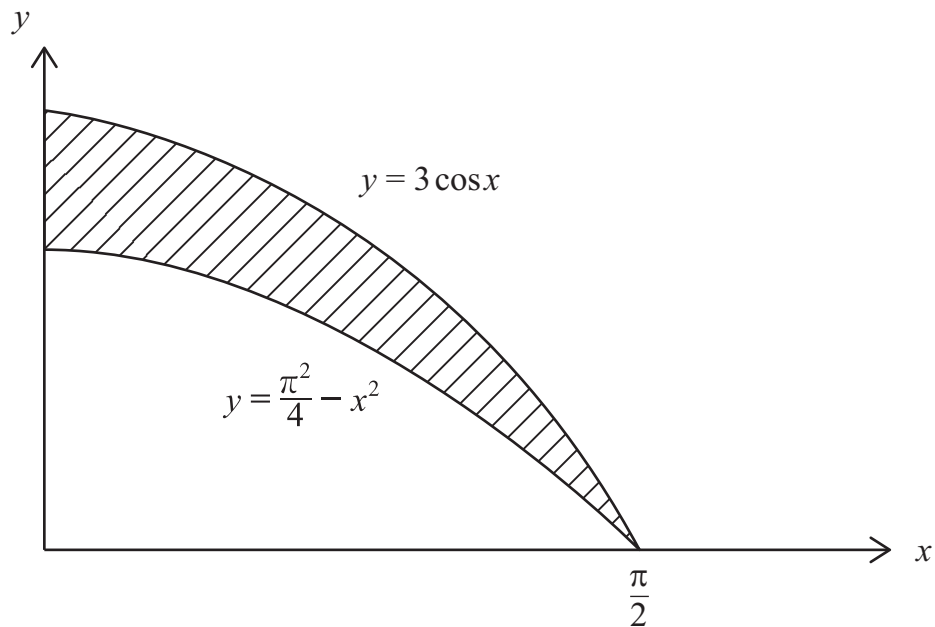


Fig. 2

Calculate the exact area of the blade.

[6]

6 (a) (i) Sketch the graph of

$$y = |\sin x|$$

$$\text{for } -360^\circ \leq x \leq 360^\circ$$

[2]

(ii) Write down a function whose graph is shown in **Fig. 3** below.

[2]

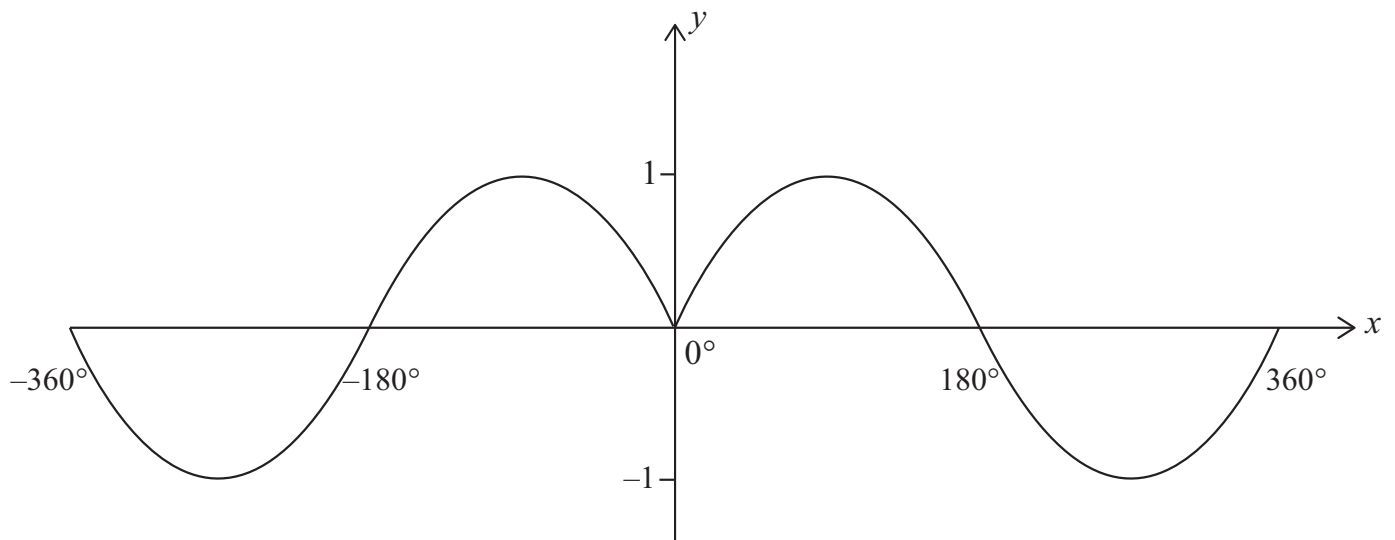


Fig. 3

(b) Solve the equation

$$\operatorname{cosec}\left(2\theta + \frac{\pi}{3}\right) = 2$$

$$\text{where } -\pi \leq \theta \leq \pi$$

[7]

7 Find the exact equation of the tangent to the curve

$$y = \tan\left(2x - \frac{\pi}{6}\right) \cos\left(x + \frac{\pi}{12}\right)$$

$$\text{at the point where } x = \frac{\pi}{4}$$

[10]

8 A potential well in nuclear physics may be modelled by the function

$$y = -\frac{\ln x}{e^x}$$

shown sketched in **Fig. 4** below:

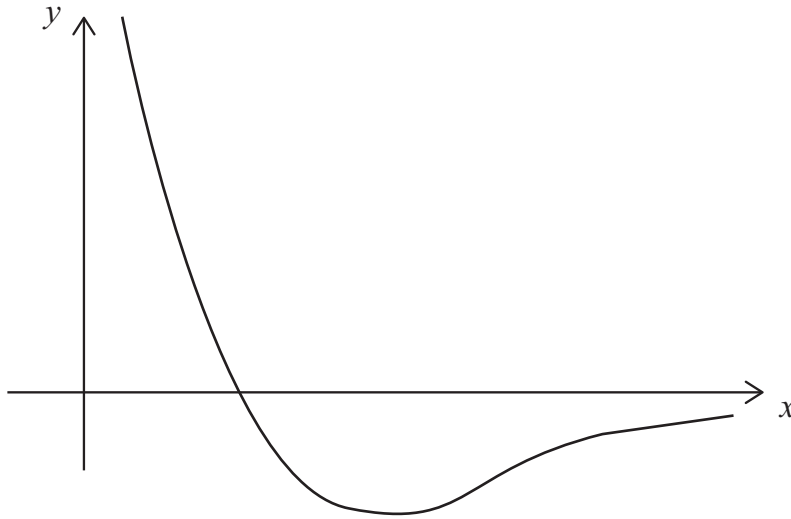


Fig. 4

(i) Find $\frac{dy}{dx}$ [3]

(ii) Show that the x coordinate of the stationary point on the curve $y = -\frac{\ln x}{e^x}$ satisfies the equation

$$\ln x = \frac{1}{x} \quad [3]$$

(iii) Hence, using the Newton–Raphson method twice with starting value 2, find an approximation to the coordinates of the stationary point. [7]

THIS IS THE END OF THE QUESTION PAPER
