Rewarding Learning

ADVANCED SUBSIDIARY (AS)
General Certificate of Education
2014

Mathematics

Assessment Unit F1

assessing

Module FP1: Further Pure Mathematics 1

[AMF11]

TUESDAY 24 JUNE, MORNING

MARK SCHEME

GCE Advanced/Advanced Subsidiary (AS) Mathematics

Mark Schemes

Introduction

The mark scheme normally provides the most popular solution to each question. Other solutions given by candidates are evaluated and credit given as appropriate; these alternative methods are not usually illustrated in the published mark scheme.

The marks awarded for each question are shown in the right hand column and they are prefixed by the letters **M**, **W** and **MW** as appropriate. The key to the mark scheme is given below:

M indicates marks for correct method.

W indicates marks for correct working.

MW indicates marks for combined method and working.

The solution to a question gains marks for correct method and marks for an accurate working based on this method. Where the method is not correct no marks can be given.

A later part of a question may require a candidate to use an answer obtained from an earlier part of the same question. A candidate who gets the wrong answer to the earlier part and goes on to the later part is naturally unaware that the wrong data is being used and is actually undertaking the solution of a parallel problem from the point at which the error occurred. If such a candidate continues to apply correct method, then the candidate's individual working must be followed through from the error. If no further errors are made, then the candidate is penalised only for the initial error. Solutions containing two or more working or transcription errors are treated in the same way. This process is usually referred to as "follow-through marking" and allows a candidate to gain credit for that part of a solution which follows a working or transcription error.

Positive marking:

It is our intention to reward candidates for any demonstration of relevant knowledge, skills or understanding. For this reason we adopt a policy of **following through** their answers, that is, having penalised a candidate for an error, we mark the succeeding parts of the question using the candidates's value or answers and award marks accordingly.

Some common examples of this occur in the following cases:

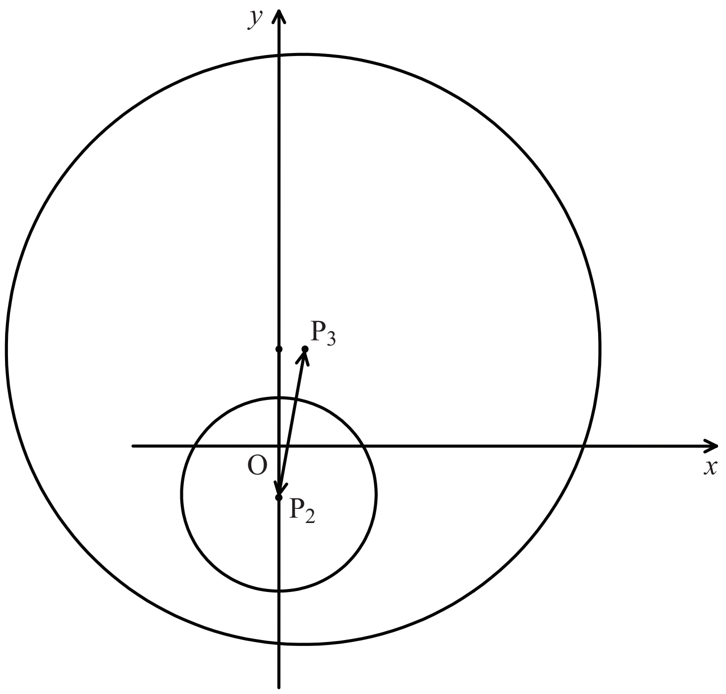
(a) a numerical error in one entry in a table of values might lead to several answers being incorrect, but these might not be essentially separate errors;

(b) readings taken from candidates' inaccurate graphs may not agree with the answers expected but might be consistent with the graphs drawn.

When the candidate misreads a question in such a way as to make the question easier only a proportion of the marks will be available (based on the professional judgement of the examining team).

			AVAILABLE MARKS
1	$\mathbf{ABC = I}$		
	$\Rightarrow \mathbf{A^{-1}\,ABC = A^{-1}\,I}$		
	$\Rightarrow \mathbf{BC = A^{-1}}$	MW1	
	$\Rightarrow \mathbf{BCC^{-1} = A^{-1}C^{-1}}$		
	$\Rightarrow \mathbf{B = A^{-1}C^{-1}}$	MW1	
	$\mathbf{A^{-1}} = \frac{1}{1}\begin{pmatrix} 3 & 5 \\ 1 & 2 \end{pmatrix} \quad \mathbf{C^{-1}} = \frac{1}{2}\begin{pmatrix} 2 & 0 \\ -4 & 1 \end{pmatrix}$	MW1 MW1	
	Hence $\mathbf{B} = \frac{1}{2}\begin{pmatrix} 3 & 5 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} 2 & 0 \\ -4 & 1 \end{pmatrix}$		
	$= \frac{1}{2}\begin{pmatrix} -14 & 5 \\ -6 & 2 \end{pmatrix}$		
	$= \begin{pmatrix} -7 & 2\frac{1}{2} \\ -3 & 1 \end{pmatrix}$	MW1	5
2	**(i)** $x^2 + y^2 + 2x - 6y - 15 = 0$ ①		
	$x^2 + y^2 + 2y - 3 = 0$ ②		
	① − ② $\Rightarrow 2x - 8y - 12 = 0$	M1	
	$\Rightarrow x = 4y + 6$	W1	
	Substitute into ②	M1	
	$\Rightarrow (4y + 6)^2 + y^2 + 2y - 3 = 0$	W1	
	$\Rightarrow 16y^2 + 48y + 36 + y^2 + 2y - 3 = 0$		
	$\Rightarrow 17y^2 + 50y + 33 = 0$	W1	
	$\Rightarrow (17y + 33)(y + 1) = 0$		
	$\Rightarrow y = -1, -\frac{33}{17}$	W1	
	$\Rightarrow x = 2, -\frac{30}{17}$	W1	
	Points of intersection are $(2, -1)$ and $\left(-\frac{30}{17}, -\frac{33}{17}\right)$	W1	

		AVAILABLE MARKS
(ii) C_2: $x^2 + (y+1)^2 = 4$		
Hence centre is (0, –1) and radius is 2	MW2	
C_3: $x^2 + y^2 - x - 4y - 31\frac{3}{4} = 0$		
$\Rightarrow (x - \frac{1}{2})^2 + (y-2)^2 = 36$		
Hence centre is $(\frac{1}{2}, 2)$ and radius is 6	MW2	



Therefore distance between centres $P_2P_3 = \sqrt{(\frac{1}{2})^2 + 3^2}$	M1	
$= \sqrt{\frac{37}{4}}$		
$= \frac{\sqrt{37}}{2}$	W1	
For C_2 to lie inside C_3 it is necessary that $P_2P_3 < r_3 - r_2$	M1	
Substituting values we have $\frac{\sqrt{37}}{2} < 6 - 2$ QED	MW1	
Hence C_2 lies entirely inside C_3		16
3 **(i)** $(a, b) * [(c, d) * (e, f)]$		
$= (a, b) * (ce, d + f + 2)$	M1	
$= (ace, b + d + f + 2 + 2)$	W1	
Also, $[(a, b) * (c, d)] * (e, f)$	M1	
$= (ac, b + d + 2) * (e, f)$		
$= (ace, b + d + f + 2 + 2)$	W1	
Hence the operation is associative.		
(ii) $(a, b) * (c, d) = (a, b)$		
$\Rightarrow (ac, b + d + 2) = (a, b)$	M1	
$\Rightarrow ac = a$ and $b + d + 2 = b$	M1	
$\Rightarrow c = 1$ $\Rightarrow d = -2$		
Hence identity = $(1, -2)$		
Check also that $(1, -2) * (a, b) = (a, -2 + b + 2) = (a, b)$	W2	8

			AVAILABLE MARKS
4	**(i)**	$\begin{vmatrix} 5 & -2 & -a \\ 4 & 2 & -6 \\ 1 & a & -4 \end{vmatrix}$	
		$= 5(-8+6a) + 2(-16+6) - a(4a-2)$ — M1	
		$= -40 + 30a - 32 + 12 - 4a^2 + 2a$ — W1	
		$= -4a^2 + 32a - 60$ — W1	

(ii) When $a = 3$, determinant $= -4(9) + 32(3) - 60 = 0$ MW1
Therefore there is not a unique solution. MW1

$5x - 2y - 3z = 3$ ① M1
$4x + 2y - 6z = 2$ ②
$x + 3y - 4z = 0$ ③

① + ② $\Rightarrow 9x - 9z = 5$ ④ MW1
3 × ② −2 × ③ $\Rightarrow 10x - 10z = 6$ ⑤
④ $\Rightarrow x - z = \frac{5}{9}$
⑤ $\Rightarrow x - z = \frac{6}{10}$

Since these equations contradict, there is no solution. MW1

When $a = 4$, determinant $= -4(16) + 32(4) - 60 = 4$ MW1
Since determinant is non-zero, there is one unique solution. MW1

(iii) $5x - 2y - 5z = 3$ ①
$4x + 2y - 6z = 2$ ②
$x + 5y - 4z = 0$ ③

① + ② $\Rightarrow 9x - 11z = 5$ ④ M1
5 × ② −2 × ③ $\Rightarrow 18x - 22z = 10$ ⑤
Since ⑤ = 2 × ④, these equations are consistent.
Re-arranging ④ $\Rightarrow x = \frac{5+11z}{9}$ MW1
Substituting into ③ $\Rightarrow \frac{5+11z}{9} + 5y - 4z = 0$ M1

$\Rightarrow 5 + 11z + 45y - 36z = 0$

$\Rightarrow 5 + 45y = 25z$
$\Rightarrow y = \frac{5z-1}{9}$ MW1 14

Hence general solution is $\left(\frac{5+11z}{9}, \frac{5z-1}{9}, z\right)$

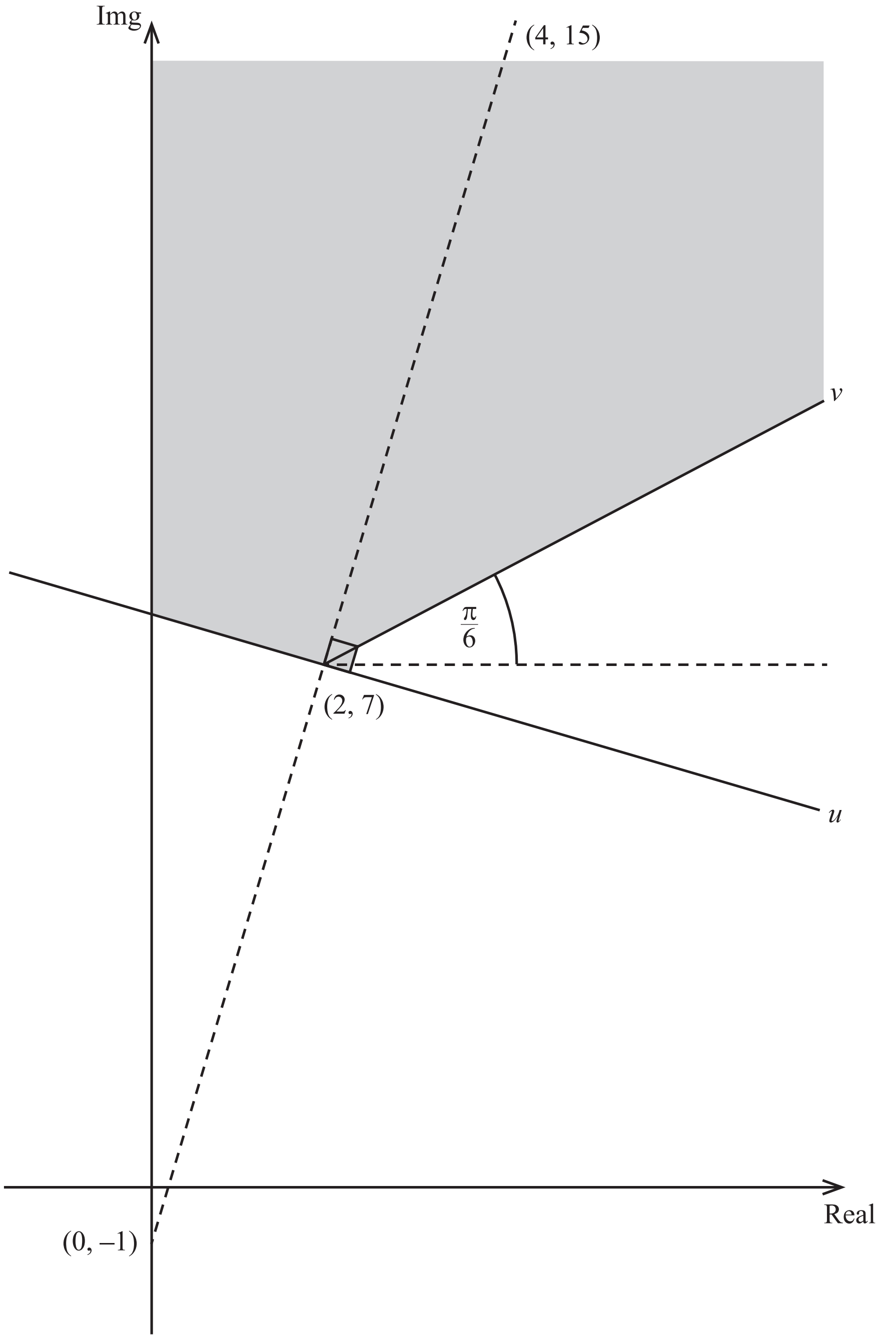
				AVAILABLE MARKS
5 **(a)** $\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 2 & 5 \\ 3 & -1 \end{pmatrix} = \begin{pmatrix} 7 & 9 \\ 2 & 5 \end{pmatrix}$				
$\Rightarrow 2a + 3b = 7$	and	$2c + 3d = 2$	M1	
$5a - b = 9$		$5c - d = 5$	M1	
$\Rightarrow 2a + 3b = 7$	and	$2c + 3d = 2$		
$15a - 3b = 27$		$15c - 3d = 15$	M1	
$\Rightarrow 17a = 34$	and	$17c = 17$		
$\Rightarrow a = 2$	and	$c = 1$	W1	
$\Rightarrow b = 1$	and	$d = 0$	W1	
Hence $\mathbf{M} = \begin{pmatrix} 2 & 1 \\ 1 & 0 \end{pmatrix}$				
(b) $\begin{pmatrix} 0 & -1 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} X \\ Y \end{pmatrix}$			M1	
$\Rightarrow -y = X$	①		MW1	
$x + 2y = Y$	②			
① $\Rightarrow y = -X$			MW1	
② $\Rightarrow x - 2X = Y$				
$\Rightarrow x = 2X + Y$			MW1	
Substitute into $x^2 + 5y^2 + 4xy - 6x - 8y = 0$			M1	
$\Rightarrow (2X + Y)^2 + 5(-X)^2 + 4(2X + Y)(-X) - 6(2X + Y) - 8(-X) = 0$			W1	
$\Rightarrow 4X^2 + 4XY + Y^2 + 5X^2 - 8X^2 - 4XY - 12X - 6Y + 8X = 0$			W1	
$\Rightarrow X^2 + Y^2 - 4X - 6Y = 0$			W1	
$\Rightarrow (X - 2)^2 + (Y - 3)^2 = 13$				
This is the equation of a circle of radius $\sqrt{13}$			MW1	14

		AVAILABLE MARKS
6 **(a)** **(i)** $z_1 = \sqrt{3} + \mathrm{i}$		
$\Rightarrow r_1 \cos\theta_1 = \sqrt{3}$	M1	
and $r_1 \sin\theta_1 = 1$		
$\Rightarrow \tan\theta_1 = \frac{1}{\sqrt{3}}$	M1	
$\Rightarrow \theta_1 = \frac{\pi}{6}$	W1	
and $r_1 = 2$	W1	
$z_2 = 1 - \mathrm{i}$		
$\Rightarrow r_2 \cos\theta_2 = 1$		
and $r_2 \sin\theta_2 = -1$		
$\Rightarrow \tan\theta_2 = -1$		
$\Rightarrow \theta_2 = -\frac{\pi}{4}$	MW1	
and $r_2 = \sqrt{2}$	MW1	
(ii) $z_1 z_2 = (\sqrt{3} + \mathrm{i})(1 - \mathrm{i})$		
$= \sqrt{3} + 1 + \mathrm{i} - \sqrt{3}\mathrm{i}$	MW1	
$\Rightarrow \lvert z_1 z_2 \rvert = \sqrt{(\sqrt{3}+1)^2 + (1-\sqrt{3})^2}$	M1	
$= \sqrt{3 + 2\sqrt{3} + 1 + 1 - 2\sqrt{3} + 3}$		
$= \sqrt{8}$		
$= 2\sqrt{2}$	W1	
$= \lvert z_1 \rvert \lvert z_2 \rvert$ QED	MW1	

(b) (i) Perpendicular bisector of line joining (4, 15) and (0, −1) MW3

(ii) Half-line through (2, 7) at angle of $\frac{\pi}{6}$ with positive x-axis MW3

AVAILABLE MARKS



(iii) Region as shown shaded in diagram above. MW2 18

Total **75**