Rewarding Learning

ADVANCED
General Certificate of Education
2014

Mathematics

Assessment Unit M4

assessing

Module M4: Mechanics 4

[AMM41]

AMM41

MONDAY 23 JUNE, MORNING

TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.
Answer **all seven** questions.
Show clearly the full development of your answers.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or a scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
Answers should include diagrams where appropriate and marks may be awarded for them.
Take $g = 9.8\,\text{m}\,\text{s}^{-2}$, unless specified otherwise.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

Answer all seven questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

1 **Fig. 1** below shows a scalene quadrilateral ABCD with AB = 2.9 m, BC = 1.2 m, CD = 1.6 m, DA = 2.1 m and the diagonal BD = 2.0 m.

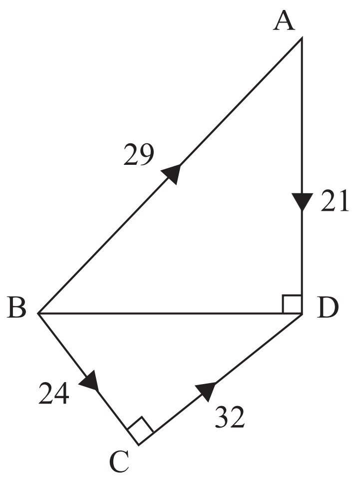

Fig. 1

Forces of 29 N, 21 N, 24 N and 32 N act along the sides BA, AD, BC and CD and in those directions respectively.
The system reduces to a single force R that crosses AD at a distance d metres above D.

Find the magnitude and direction of R and the value of d. [8]

2 **Fig. 2** below shows the jib of a crane modelled by a framework of five light pin jointed rods AB, BC, CA, CD and DA. The jib is freely hinged at the point D to a fixed point at the top of a rigid steel tower.

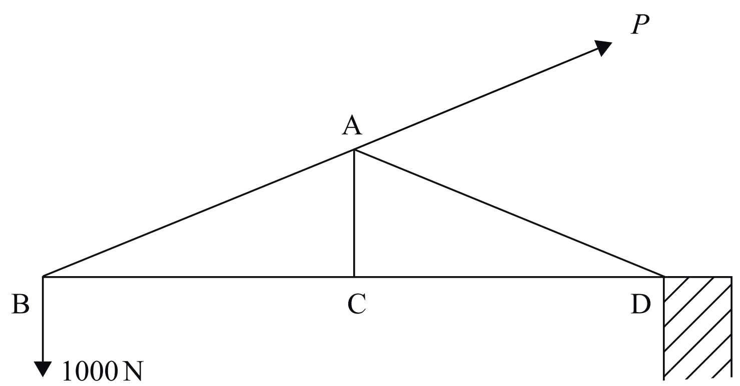

Fig. 2

The jib carries a load of 1000 N at B. It is held in equilibrium, with BD horizontal, by a cable attached at A, which exerts a force P newtons at 30° above the horizontal.

$\mathrm{AB} = \mathrm{AD} = 2l$, $\mathrm{AC} = l$ and $\mathrm{BC} = \mathrm{CD} = \sqrt{3}l$ metres.

(i) Find P. [3]

(ii) Explain why the force in BA is a tension and that in BC is a thrust. [2]

(iii) Find the forces in BA and BC respectively. [3]

(iv) Explain why no force exists in AC or in AD in this situation. [2]

3 A particle of mass m is moving in a vertical circle on the smooth outside surface of a cylinder. The circle has radius r and vertical diameter AOB.
The centre of the circle is O and A is above B.
When the particle is at P its speed is v and the angle AOP is θ as shown in **Fig. 3** below.

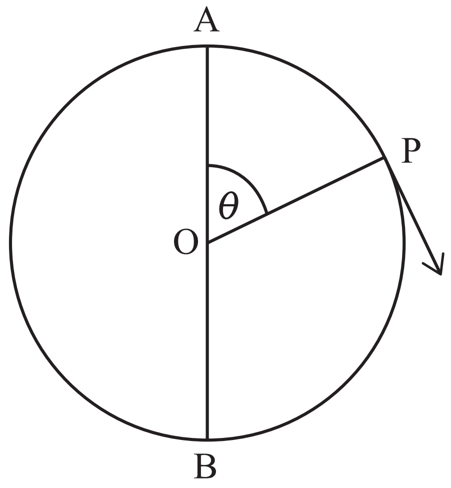

Fig. 3

The particle moves from rest at A.
Take the gravitational potential energy to be zero at B.

(i) Find in terms of m, g, r and θ an expression for the kinetic energy of the particle when it reaches P. [4]

(ii) Show that the reaction, R, of the surface at P on the particle is given by

$$R = mg(3\cos\theta - 2)$$ [5]

The particle leaves the surface and moves freely as a projectile after OP moves through a critical angle θ_c

(iii) Find θ_c [2]

4 A particle of weight mg lies on the surface of the Earth which has mass M and radius r.

(i) Find in terms of g, M and r an expression for the Universal Gravitational Constant, G, and hence show that its dimensions are $[\mathrm{M}^{-1}\mathrm{L}^{3}\mathrm{T}^{-2}]$. [3]

In the distant future it is believed that a new celestial body, S, will become trapped in an orbit centered on the Earth. If the orbit is circular, of radius d, then the orbital period T is expected to take the form

$$T = kM^{x}d^{y}\mathrm{G}^{z} \qquad \text{where } k \text{ is a dimensionless constant.}$$

(ii) Use the method of dimensions to find x, y and z. [5]

(iii) By considering S as a satellite of mass m_1 moving in a circle around the Earth with constant angular velocity ω, verify your result in **(ii)** and find the exact value of k. [5]

5 A particle A of mass km is moving with constant speed u along a straight horizontal path directly towards a second particle B of mass m that is at rest.
A collides with B.
After the collision A and B move with velocities v_1 and v_2 respectively.
The coefficient of restitution between the particles is e.

(i) Find v_1 and show that $v_2 = \dfrac{(1+e)ku}{(1+k)}$ [6]

B then hits a wall perpendicular to its path and rebounds with speed v_3
The coefficient of restitution between B and the wall is also e.

(ii) Find v_3 and explain briefly why there will be another collision if $k > e$. [2]

(iii) If $k < e$, show that there will be another collision if

$$ke^{2} - (1-k)e + k > 0$$ [3]

6 The rudder for a boat is made from a metal lamina cut in the shape of the area contained between the curve

$$y_1 = x(3 - x)$$

and the straight line

$$y_2 = x$$

The density of the metal lamina is p kg m^{-2} and the distances are measured in metres. **Fig. 4** below shows a design drawing with an infinitesimally thin strip of thickness δx drawn parallel to the y-axis.

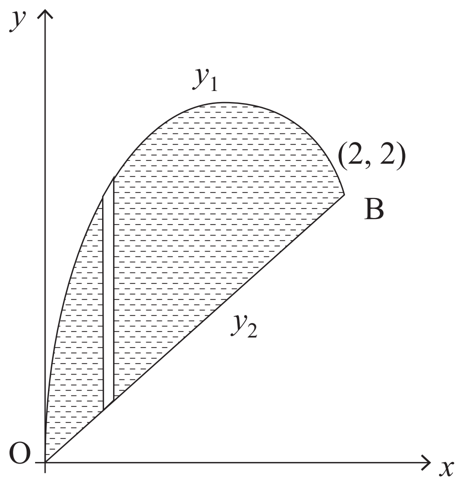

Fig. 4

To find the position of the centre of mass of the rudder the following integrals need to be evaluated:

Mass $\quad \mathrm{I}_1 = \int_0^2 p(y_1 - y_2)\,\mathrm{d}x$

Moment about the y-axis $\quad \mathrm{I}_2 = \int_0^2 px(y_1 - y_2)\,\mathrm{d}x$

Moment about the x-axis $\quad \mathrm{I}_3 = \int_0^2 \tfrac{1}{2}p(y_1 + y_2)(y_1 - y_2)\,\mathrm{d}x$

(i) Explain briefly the significance of the expressions $(y_1 - y_2)$ and $\tfrac{1}{2}(y_1 + y_2)$ in the integrands of I_2 and I_3 respectively. [2]

(ii) If $\mathrm{I}_1 = \dfrac{4}{3}p$, calculate I_2 and hence show that the x-coordinate of G, the centre of mass of the rudder, is 1 [4]

(iii) If $\mathrm{I}_3 = \dfrac{28}{15}p$, find the y-coordinate of G. [1]

The length of the straight edge of the rudder is $2\sqrt{2}$ metres and the weight of the rudder is 250 N.

(iv) Show that G is $0.2\sqrt{2}$ metres from this straight edge. [2]

The rudder will be fitted to the boat with the straight edge OB vertical. Two hinges will freely hold it at B and at the mid-point, M, of OB as shown in **Fig. 5** below.

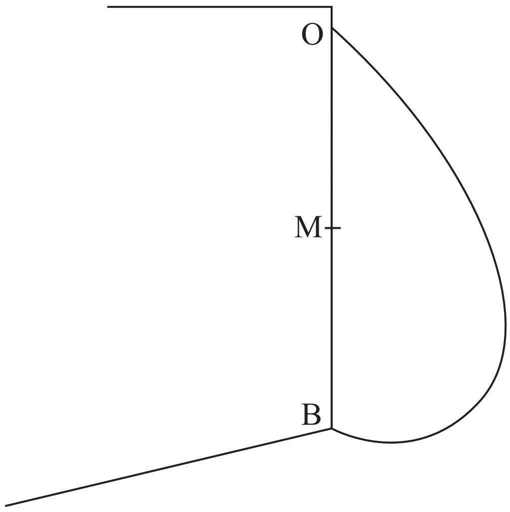

Fig. 5

After the hinge has been fitted at B, the rudder must be held in place so that the second hinge can be fitted at M.

(v) Find the minimum force needed to hold the rudder with OB in its vertical position while the second hinge is being fitted. [3]

7 Axel is studying the forces acting on a car of mass m moving at speed v in a horizontal circle of radius r as it rounds a very dangerous bend banked downwards at an angle α to the horizontal.

His diagram – shown in **Fig. 6** below – indicates the normal and frictional reactions of the road surface on the car together with the weight. The car is just about to slip down the banking.

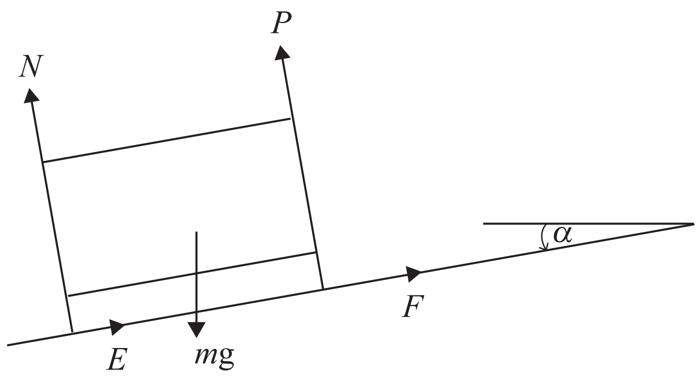

Fig. 6

The coefficient of friction between the car and the road is μ.

(i) Show that

$$v^2 = rg\left(\frac{\mu - \tan\alpha}{\mu\tan\alpha + 1}\right)$$ [8]

The radius r of the bend is 10 m, $\mu = \frac{6}{7}$ and $\tan\alpha = \frac{1}{4}$

(ii) Find v. [2]

THIS IS THE END OF THE QUESTION PAPER