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ADVANCED
General Certificate of Education
2014

Mathematics

Assessment Unit M3

assessing

Module M3: Mechanics 3

[AMM31]

AMM31

MONDAY 16 JUNE, MORNING

TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.
Answer **all seven** questions.
Show clearly the full development of your answers.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
Answers should include diagrams where appropriate and marks may be awarded for them.
Take $g = 9.8\,\text{m}\,\text{s}^{-2}$, unless specified otherwise.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

Answer all seven questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

1 **Fig. 1** below shows two light inextensible strings AB and PC used to keep a small smooth ring hanging in equilibrium. AB passes through the ring and PC is attached to the ring at P. A is attached to a fixed point on a vertical wall and B is attached to a fixed point on a horizontal ceiling. The ring can be modelled as a particle of weight 40 N.

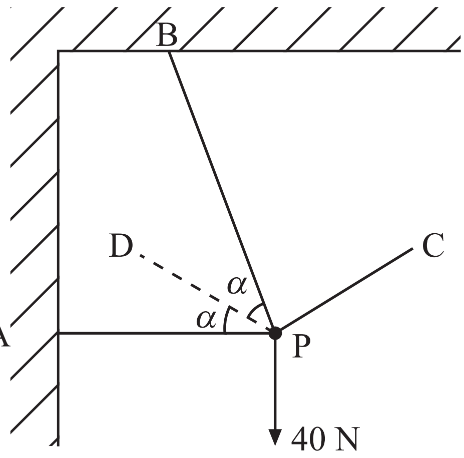

Fig. 1

AP is horizontal and the angle APB is 2α, where $\cos\alpha = 0.8$
PD bisects the angle APB.

(i) Explain briefly why the resultant, R, of the forces in AP and PB acts along PD. [1]

The tension in AP is S newtons.

(ii) Find, in terms of S, the magnitude of R. [2]

The tension, T, in PC is a minimum when PC makes an angle θ with the horizontal.

(iii) Explain briefly why $\sin\theta = 0.8$ [2]

(iv) Find the minimum value of T. [3]

2 The forces $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ newtons act on a particle, P, moving it along the line whose vector equation is

$$\mathbf{r} = \lambda\mathbf{b}$$

where λ is a scalar constant and $\mathbf{b}$ is measured in metres.

Given that

$$\mathbf{F}_1 = \begin{pmatrix} 3 \\ -5 \\ 4 \end{pmatrix} \quad \text{and} \quad \mathbf{b} = \begin{pmatrix} 3 \\ 5 \\ 4 \end{pmatrix}$$

(i) show that the work done by $\mathbf{F}_1$ as P is moved along the line is 0 J. [2]

(ii) Explain what the result in **(i)** says about the directions of $\mathbf{F}_1$ and $\mathbf{b}$. [1]

A and B are the two points on the line where λ takes the values 0 and 2 respectively. The total work done by the three forces as the particle moves from A to B is W joules.

Given that

$$\mathbf{F}_2 = \begin{pmatrix} 8 \\ 2a \\ 9 \end{pmatrix} \quad \text{and} \quad \mathbf{F}_3 = \begin{pmatrix} -a \\ a \\ -a \end{pmatrix}$$

where a is a scalar constant,

(iii) find W in terms of a. [4]

(iv) If $W = 200$, find a. [1]

3 An engineering component is made from a rectangular metal lamina ABCD of width 32 cm and height 60 cm.
Fig. 2 below shows the first stage of production where an isosceles triangle EHF of base 16 cm and sides 17 cm is cut from the lamina with BE = FC = 8 cm.

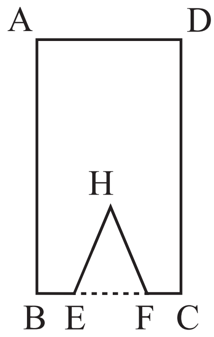

Fig. 2

(i) If the mass of metal that is removed is 0.2 kg show that the mass remaining is 3 kg. [3]

(ii) Show that the centre of mass at the end of this first stage is $1\frac{2}{3}$ cm above the centre of the original lamina ABCD. [5]

(iii) If the lamina is freely suspended from A, find the angle that AB makes with the vertical. [3]

At the second stage of production the corners at A and D are removed by cutting off two identical right angled triangles with perpendicular sides 8 cm and 15 cm.
The right angles in the triangles coincide with the right angles at A and D.
The centre of mass is now at the centre of the original lamina ABCD.
Fig. 3 below shows the correct shape of the component after the second stage of production together with an incorrect shape.

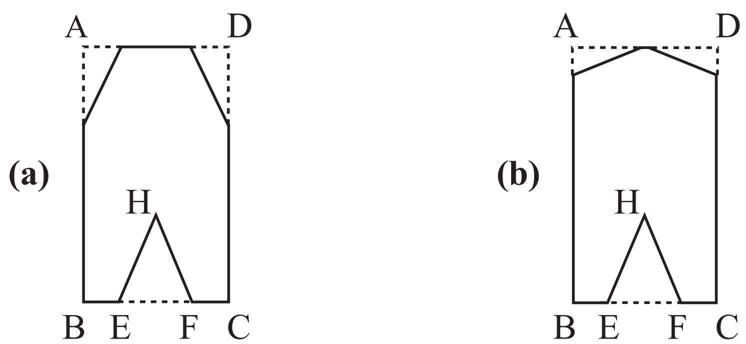

Fig. 3

(iv) State which shape is the correct one, giving a brief explanation for your choice. [2]

4 The X-potential motor has been developed by aliens for a new spacecraft.
The thrust $T(x)$ meganewtons (i.e. 10^6 newtons) produced by the motor is given by

$$T(x) = e^{x-1} + e^{1-x}$$

where x metres is the distance travelled from rest.
The motor is used to propel a two tonne spacecraft from a position of rest in gravity free space but needs to act only for the first two metres of travel.

(i) Show that the work done by the motor over the first 2 m travelled by the spacecraft is approximately 4.70×10^6 J. [5]

(ii) Find the speed of the spacecraft when it has travelled the first two metres. [2]

5 A particle P of mass 2 kg is attached to one end of a light elastic string of natural length 5 m and modulus of elasticity 16g N. The other end of the string is attached to a fixed point A on a rough horizontal surface.
P is held at a point B on the surface with the string just taut. It is then given a speed of $21\,\text{m}\,\text{s}^{-1}$ in the direction AB.

(i) Evaluate, in terms of d, the work done on the particle by the tension in the string as P moves a distance d metres beyond B. [2]

The coefficient of friction between the particle and the surface is 0.5

(ii) Use the Work–Energy Principle to find how far P moves beyond B, before it comes to rest. [6]

(iii) Determine whether this position of rest is instantaneous or permanent. [2]

6 Take Glasgow airport to be 200 km NE of Belfast airport.
Planes on the Belfast–Glasgow route operate at 400 km h^{-1} in still air.
One day the wind was blowing from W15° N at 50 km h^{-1}
The pilot set the course on the Belfast–Glasgow route at $\theta°$ anti-clockwise from its intended flight path as shown in **Fig. 4** below.

Fig. 4

(i) Explain briefly why the plane's course was set in this direction. [1]

The plane covered the flight path at v km h^{-1}

(ii) By considering the velocity diagram for the flight show that

$$v^2 - 50v - 157\,500 = 0 \qquad \text{equation (a)}$$ [5]

(iii) Find the time taken to fly from Belfast to Glasgow. [2]

The plane was unable to land at Glasgow and immediately had to return to Belfast.
The return flight path was covered at w km h^{-1}

(iv) By considering the velocity diagram for this flight show that

$$w^2 + 50w - 157\,500 = 0 \qquad \text{equation (b)}$$ [3]

(v) Show that if $w = -v$ then equation (b) transforms to equation (a). [1]

The plane left Belfast at noon.

(vi) Find the time at which it landed back in Belfast. [2]

7 **Fig. 5** below shows a particle of mass m attached to one end of each of two elastic strings, S_1 and S_2
The other ends of the strings are attached to fixed points on a smooth horizontal surface – S_1 to A and S_2 to B.

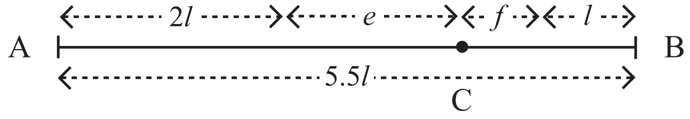

Fig. 5

S_1 has natural length $2l$ and modulus of elasticity λ.
S_2 has natural length l and modulus of elasticity 2λ.
The distance AB is $5.5l$
When the particle is in equilibrium at C the extension in S_1 is e and the extension in S_2 is f.

(i) Show that $e = 2l$ [5]

The particle is displaced a small distance d to the right of C so that both strings remain taut. It is released from rest where $d < 0.25l$
At time t its displacement from C is x.

(ii) Show that the equation of motion, in terms of x, is of the standard form for S.H.M. [5]

(iii) Given that $\lambda = 6.4\,ml\pi^2$, show that the period of the motion is 0.5 [2]

The particle is released from rest at $t = 0$ and $x = 0.2l$

(iv) Write down in its simplest form the expression for x as a function of time. [1]

(v) Briefly explaining your answer, state the maximum distance the particle could be displaced to the right of C and still move in S.H.M. [2]

THIS IS THE END OF THE QUESTION PAPER