



Rewarding Learning

ADVANCED  
General Certificate of Education  
January 2014

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## Mathematics

### Assessment Unit C4

*assessing*

Module C4: Core Mathematics 4

[AMC41]

WEDNESDAY 22 JANUARY, MORNING

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#### TIME

1 hour 30 minutes.

#### INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.

Answer **all eight** questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

You are permitted to use a graphic or scientific calculator in this paper.

#### INFORMATION FOR CANDIDATES

The total mark for this paper is 75

Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.

A copy of the **Mathematical Formulae and Tables booklet** is provided.

Throughout the paper the logarithmic notation used is  $\ln z$  where it is noted that  $\ln z \equiv \log_e z$

Answer all eight questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

- 1 Use the substitution  $u = 2x - 3$  to find

$$\int 4x(2x - 3)^5 dx \quad [6]$$

- 2 **Fig. 1** below shows a vertical tower TO of height 25 m. The tower is held in place by three cables joined to the top, T, of the tower. The cables are fastened to the ground at A, B and C. Taking the tower to lie along the z-axis and the base of the tower, O, to be the origin (0, 0, 0), the points A, B and T have position vectors:

$$\vec{OA} = \begin{pmatrix} 2 \\ -6 \\ -1 \end{pmatrix} \quad \vec{OB} = \begin{pmatrix} 2 \\ 6 \\ 1 \end{pmatrix} \quad \vec{OT} = \begin{pmatrix} 0 \\ 0 \\ 25 \end{pmatrix}$$

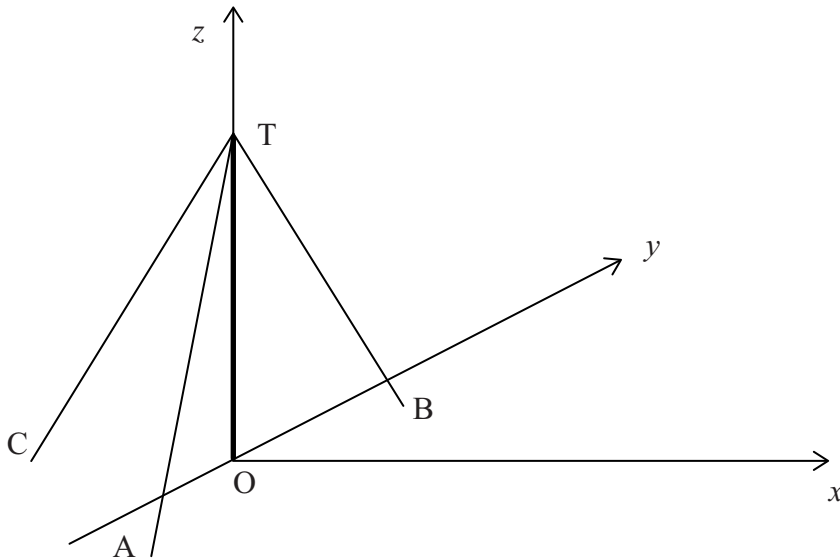


Fig. 1

- (i) Find the lengths of the cables AT and BT. [5]
- (ii) Find the angle between the cables AT and BT. [4]

3 Find the **exact** area bounded by the curve

$$y = \frac{1 - 10x}{(2x + 1)(1 - x)}$$

the  $x$ -axis and the lines  $x = 2$  and  $x = 3$ , using partial fractions.

[12]

4 Given that

$$x^2y - x + 1 = 0$$

use implicit differentiation to show that

$$2y + 4x \frac{dy}{dx} + x^2 \frac{d^2y}{dx^2} = 0$$

[9]

5 The functions  $f$  and  $g$  are defined as:

$$\begin{aligned} f: x &\rightarrow e^x & x &\in \mathbb{R} \quad x \geq 0 \\ g: x &\rightarrow 5x + 2 & x &\in \mathbb{R} \end{aligned}$$

(i) State the range of  $f$ .

[1]

(ii) Find the composite function  $gf$  and state its domain and range.

[5]

(iii) Find the **exact** value of  $x$  such that  $gf(x) = 11$

[2]

6 Solve the differential equation

$$\frac{dy}{dx} = \frac{\operatorname{cosec} 3y}{\tan x}$$

given that  $x = \frac{\pi}{2}$  when  $y = \frac{\pi}{3}$

[8]

7 (i) Prove the identity

$$\cos 3x \equiv 4 \cos^3 x - 3 \cos x \quad [6]$$

(ii) Hence solve the equation

$$12 \cos^2 x - \sec x = 9 \quad 0 \leq x \leq \pi \quad [6]$$

8 A paperweight can be modelled as the solid formed when the area bounded by the curve

$$y = \frac{\sqrt{\ln x}}{x}$$

the  $x$ -axis and the line  $x = 5$  is rotated through  $2\pi$  radians about the  $x$ -axis.

Find the **exact** volume of the paperweight. [11]

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**THIS IS THE END OF THE QUESTION PAPER**

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