Rewarding Learning

ADVANCED SUBSIDIARY (AS)
General Certificate of Education
January 2014

Mathematics

Assessment Unit F1

assessing

Module FP1: Further Pure Mathematics 1

[AMF11]

AMF11

FRIDAY 10 JANUARY, MORNING

TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.
Answer **all six** questions.
Show clearly the full development of your answers.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

Answer all six questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

1 Let $\mathbf{A} = \begin{pmatrix} 1 & x & 2 \\ 2 & 3 & 0 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 0 & 6 \\ 1 & x \\ -1 & -3 \end{pmatrix}$

(i) Calculate the matrix product **AB**, leaving your answer in terms of x. [2]

(ii) Given that **AB** has no inverse, find the value of x. [4]

2 **(a)** Write down the 2×2 matrix **M** which represents a reflection in the line

$$y = \frac{1}{\sqrt{3}}x$$ [3]

(b) The matrix $\mathbf{N} = \begin{pmatrix} 1 & 2 \\ 4 & -1 \end{pmatrix}$ represents a linear transformation of the x–y plane.

Find the equations of the straight lines through the origin which are invariant under the transformation given by **N** [6]

3 **(a)** The set of numbers $\{1, 3, 7, a\}$, where a is a positive integer, forms a group under multiplication modulo 8
Find the value of a. [1]

(b) **(i)** Copy and complete the following group table for multiplication modulo 18

$\times_{18}$	1	5	7	11	13	17
1	1	5	7	11	13	17
5	5	7	17	1	11	13
7	7	17	13	5	1	11
11	11	1	5			
13	13	11	1			
17	17	13	11			

[3]

(ii) Find an element which generates this group, briefly justifying your answer. [2]

(iii) Write down a subgroup of this group of order 2 [2]

(iv) Write down a subgroup of this group of order 3 [2]

4 The matrix **P** is given by

$$\mathbf{P} = \begin{pmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{pmatrix}$$

(i) Show that the eigenvalues of **P** are 2, 3 and 6 [7]

(ii) For the eigenvalue 6, find a corresponding eigenvector. [4]

(iii) Verify that $\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ are eigenvectors of **P** [4]

(iv) If $\mathbf{U}^{\mathrm{T}}\mathbf{P}\mathbf{U} = \mathbf{D}$, where **D** is a diagonal matrix, write down a possible matrix **U** [1]

5 Two circles have equations

$$x^2 + y^2 + 6x - 10y + 9 = 0$$
$$x^2 + y^2 - 2x - 6y + 1 = 0$$

(i) Find the centre and radius of each circle. [4]

(ii) Determine whether or not the circles intersect. Clearly justify your answer. [4]

(iii) Find the point of intersection of the common tangents to the two circles. [5]

6 **(a)** The complex number z is defined as $z = \dfrac{3 + \mathrm{i}}{1 - \mathrm{i}}$

Calculate $z + \dfrac{1}{z}$, leaving your answer in the form $a + b\mathrm{i}$, where a and b are rational numbers. [6]

(b) Given that $(p + q\mathrm{i})^2 = 17 - 6\sqrt{2}\,\mathrm{i}$, find the values of p and the corresponding values of q. [8]

(c) **(i)** Sketch on an Argand diagram the locus of those points u which satisfy

$$|u - (1 - \mathrm{i})| = 2$$ [3]

(ii) On the same Argand diagram sketch the locus of those points v which satisfy

$$|v - (4 + 5\mathrm{i})| = 1$$ [1]

(iii) Find the maximum value of $|u - v|$ where u and v are the complex numbers which satisfy the equations in **(i)** and **(ii)** respectively.

A solution by scale drawing will not be accepted. [3]

THIS IS THE END OF THE QUESTION PAPER