Rewarding Learning

ADVANCED
General Certificate of Education
January 2014

Mathematics

Assessment Unit C4

assessing

Module C4: Core Mathematics 4

[AMC41]

WEDNESDAY 22 JANUARY, MORNING

AMC41

TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.
Answer **all eight** questions.
Show clearly the full development of your answers.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

Answer all eight questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

1 Use the substitution $u = 2x - 3$ to find

$$\int 4x(2x-3)^5 \, dx \qquad [6]$$

2 **Fig. 1** below shows a vertical tower TO of height 25 m.
The tower is held in place by three cables joined to the top, T, of the tower.
The cables are fastened to the ground at A, B and C.
Taking the tower to lie along the z-axis and the base of the tower, O, to be the origin (0, 0, 0), the points A, B and T have position vectors:

$$\overrightarrow{OA} = \begin{pmatrix} 2 \\ -6 \\ -1 \end{pmatrix} \quad \overrightarrow{OB} = \begin{pmatrix} 2 \\ 6 \\ 1 \end{pmatrix} \quad \overrightarrow{OT} = \begin{pmatrix} 0 \\ 0 \\ 25 \end{pmatrix}$$

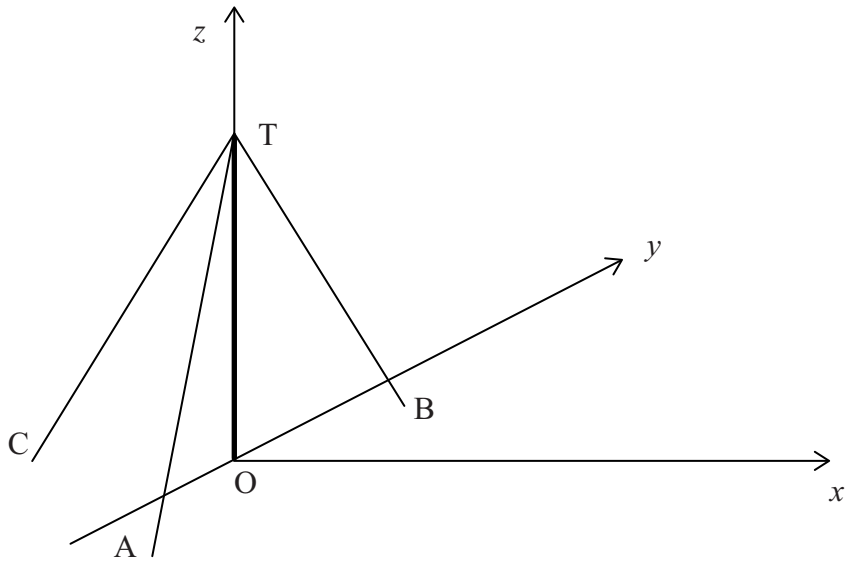

Fig. 1

(i) Find the lengths of the cables AT and BT. [5]

(ii) Find the angle between the cables AT and BT. [4]

3 Find the **exact** area bounded by the curve

$$y = \frac{1 - 10x}{(2x + 1)(1 - x)}$$

the x-axis and the lines $x = 2$ and $x = 3$, using partial fractions. [12]

4 Given that

$$x^2y - x + 1 = 0$$

use implicit differentiation to show that

$$2y + 4x\frac{dy}{dx} + x^2\frac{d^2y}{dx^2} = 0$$ [9]

5 The functions f and g are defined as:

$$\begin{array}{ll} \text{f}: x \to e^x & x \in \mathbb{R} \quad x \geqslant 0 \\ \text{g}: x \to 5x + 2 & x \in \mathbb{R} \end{array}$$

(i) State the range of f. [1]

(ii) Find the composite function gf and state its domain and range. [5]

(iii) Find the **exact** value of x such that $\text{gf}(x) = 11$ [2]

6 Solve the differential equation

$$\frac{dy}{dx} = \frac{\text{cosec } 3y}{\tan x}$$

given that $x = \frac{\pi}{2}$ when $y = \frac{\pi}{3}$ [8]

7 **(i)** Prove the identity

$$\cos 3x \equiv 4\cos^3 x - 3\cos x \qquad [6]$$

(ii) **Hence** solve the equation

$$12\cos^2 x - \sec x = 9 \qquad 0 \leqslant x \leqslant \pi \qquad [6]$$

8 A paperweight can be modelled as the solid formed when the area bounded by the curve

$$y = \frac{\sqrt{\ln x}}{x}$$

the x-axis and the line $x = 5$ is rotated through 2π radians about the x-axis.

Find the **exact** volume of the paperweight. [11]

THIS IS THE END OF THE QUESTION PAPER