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ADVANCED
General Certificate of Education
2013

Mathematics

Assessment Unit F3

assessing

Module FP3: Further Pure Mathematics 3

[AMF31]

AMF31

FRIDAY 24 MAY, MORNING

TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.
Answer **all seven** questions.
Show clearly the full development of your answers.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

Answer all seven questions

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

1 Find in Cartesian form an equation of the line of intersection of the planes

$$2x + 3y + z = 19$$

$$\text{and} \qquad \mathbf{r} \cdot \begin{pmatrix} 3 \\ 2 \\ -6 \end{pmatrix} = 16$$

[6]

2 Differentiate

$$\tan^{-1}\left(\frac{x}{\sqrt{4-x^2}}\right) + \cos^{-1}\left(\frac{x}{2}\right)$$

simplifying your answer as far as possible. [9]

3 Find

$$\int \frac{3-5y^3}{\sqrt{1-y^2}}\,\mathrm{d}y$$

[5]

4 **(i)** Using the exponential definitions of $\cosh x$ and $\sinh x$, prove the identity

$$\mathrm{e}^{nx} \equiv (\cosh x + \sinh x)^n$$

[2]

(ii) Using part **(i)** and the corresponding identity

$$\mathrm{e}^{-nx} \equiv (\cosh x - \sinh x)^n$$

prove that

$$\sinh 3x \equiv 4\sinh^3 x + 3\sinh x$$

[6]

5 **(i)** Use the exponential definition of sinh x to show that

$$\sinh^{-1}(x) \equiv \ln\left(x + \sqrt{x^2 + 1}\right)$$ [4]

(ii) Hence show that

$$\frac{\mathrm{d}}{\mathrm{d}x}(x \sinh^{-1}(x)) = \sinh^{-1} x + \frac{x}{\sqrt{x^2+1}}$$ [6]

(iii) Hence, or otherwise, show that

$$\int_0^{\frac{4}{3}} \ln\left(x + \sqrt{x^2+1}\right) \mathrm{d}x = \frac{2}{3}(2 \ln 3 - 1)$$ [7]

6 The Kiltough Jewel can be modelled by the irregular tetrahedron PQRS shown in **Fig. 1** below.

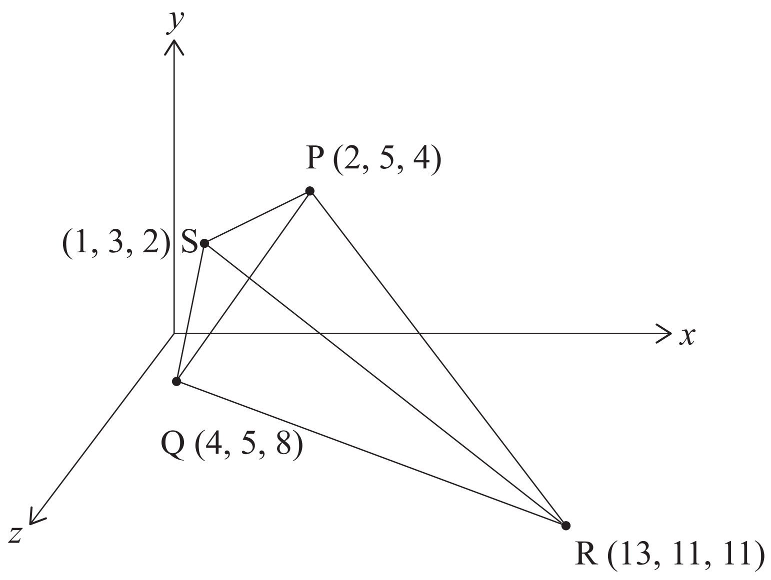

Fig. 1

(i) Derive a Cartesian equation of the plane SPQ. [7]

(ii) Using a formula of the form

$$V = \frac{1}{6}|\mathbf{a} \,.\, \mathbf{b} \times \mathbf{c}|$$

find the volume of the jewel. [4]

(iii) Calculate the angle between the edge SR and the face SPQ. [5]

7 **(i)** Differentiate

$$(1 - x^2)^{\frac{3}{2}}$$ [2]

(ii) Hence show that if

$$I_n = \int_0^1 x^n \sqrt{1 - x^2}\, \mathrm{d}x$$

for each non-negative integer n, then

$$I_n = \frac{n - 1}{n + 2} I_{n-2} \quad n \geqslant 2$$ [8]

(iii) Show that

$$\int_0^1 x^5 \sqrt{1 - x^2}\, \mathrm{d}x = \frac{8}{105}$$ [4]

THIS IS THE END OF THE QUESTION PAPER