Rewarding Learning

ADVANCED
General Certificate of Education
2013

Mathematics

Assessment Unit F2

assessing

Module FP2: Further Pure Mathematics 2

[AMF21]

AMF21

THURSDAY 30 MAY, MORNING

TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.
Answer **all seven** questions.
Show clearly the full development of your answers.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

Answer all seven questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

1 **(i)** Show that

$$\sum_{r=1}^{n}(2r-1)^2 = \frac{n}{3}(4n^2-1)$$

[4]

(ii) Using this result, evaluate

$$\sum_{r=5}^{40}(2r-1)^2$$

[3]

2 Find, in terms of π, the general solution of the equation

$$\tan^4 x - 4\tan^2 x + 3 = 0$$

[6]

3 **(i)** Using Maclaurin's theorem, show that a series expansion for $\ln(1+ax)$, where a is a constant, up to and including the term in x^3 is

$$\ln(1+ax) = ax - \frac{a^2x^2}{2} + \frac{a^3x^3}{3}$$

[6]

(ii) Hence or otherwise find a series expansion for

$$\ln\left\{\frac{(1-3x)^2}{1+2x}\right\}$$

up to and including the term in x^3 [5]

(iii) For the series expansion in **(ii)** find the coefficient of x^n [2]

4 A man adds 10 grams of sugar to a cup of hot tea. The mass of sugar, M grams, undissolved in the tea t seconds after being added is modelled by the differential equation

$$\frac{dM}{dt} + \frac{2M}{10 - t} = \frac{1}{5} \qquad 0 \leqslant t < 10$$

(i) Find M in terms of t. [9]

(ii) Find the mass of sugar undissolved after 5 seconds. [2]

5 Use the principle of mathematical induction to show that for $n \geqslant 1$

$$\sum_{r=1}^{n} r^2 \geqslant \frac{n(n+1)^2}{4}$$ [8]

6 **(i)** Show that an equation of the tangent to the parabola $y^2 = 4ax$ at the point P $(at^2, 2at)$ is

$$x - ty + at^2 = 0$$ [4]

(ii) Find the coordinates of the point Q where this tangent cuts the y-axis. [2]

(iii) If O is the origin find the point of intersection of the perpendicular bisectors of the lines PQ and OQ. [5]

(iv) As t varies find the equation of the locus of this point of intersection. [3]

7 **(i)** Using De Moivre's theorem, show that if $z = \cos\theta + \mathrm{i}\sin\theta$ then

$$2\cos n\theta = z^n + \frac{1}{z^n}$$ [3]

(ii) Hence or otherwise show that if $z = \cos\theta + \mathrm{i}\sin\theta$ the equation

$$3z^4 - 5z^3 + 8z^2 - 5z + 3 = 0$$

can be transformed into the equation

$$6\cos^2\theta - 5\cos\theta + 1 = 0$$ [4]

(iii) Using **(ii)** or otherwise solve the equation

$$3z^4 - 5z^3 + 8z^2 - 5z + 3 = 0$$ [9]

THIS IS THE END OF THE QUESTION PAPER