

ADVANCED General Certificate of Education 2012

# **Mathematics**

Assessment Unit C4 assessing Module C4: Core Mathematics 4

## [AMC41]

## FRIDAY 1 JUNE, MORNING

## TIME

1 hour 30 minutes.

## **INSTRUCTIONS TO CANDIDATES**

Write your Centre Number and Candidate Number on the Answer Booklet provided. Answer **all eight** questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

You are permitted to use a graphic or scientific calculator in this paper.

## INFORMATION FOR CANDIDATES

The total mark for this paper is 75

Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.

A copy of the Mathematical Formulae and Tables booklet is provided.

Throughout the paper the logarithmic notation used is  $\ln z$  where it is noted that  $\ln z \equiv \log_e z$ 



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#### Answer all eight questions.

#### Show clearly the full development of your answers.

#### Answers should be given to three significant figures unless otherwise stated.

- 1 (a) For each relation below state if it is a one-one function, a many-one function or a mapping.
  - (i)  $a: x \to \pm \sqrt{x} \qquad x \in \mathbb{R} \quad x \ge 0$  [1]
  - (ii)  $b: x \to \frac{1}{x} \qquad x \in \mathbb{R} \quad x \neq 0$  [1]
  - (iii)  $c: x \to x^2 \qquad x \in \mathbb{R}$  [1]
  - (b) The function f is defined by

 $\mathbf{f}(x) = x^2 - 3 \qquad x \in \mathbb{R}$ 

and the function g is defined by

$$g(x) = 2x + 1 \qquad x \in \mathbb{R}$$

- (i) Find the composite function gf(x), stating its domain. [3]
- (ii) Find the values of x such that gf(x) = 3x [3]
- 2 (i) Sketch the graph of

$$v = 4 - x^2$$

clearly showing where it crosses the x-axis.

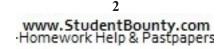
A paperweight can be modelled as the solid formed when the area bounded by the curve

$$y = 4 - x^2$$

and the x-axis and the y-axis is rotated through  $360^{\circ}$  about the x-axis.

(ii) Find the volume of the paperweight.

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[7]

[2]

**3** During a science experiment, students create two waves in a ripple tank. The first wave can be modelled by the equation

 $h_1 = 6\cos t \qquad 0 \le t \le 2\pi$ 

The second wave can be modelled by the equation

$$h_2 = 5 \sin t$$
  $0 \le t \le 2\pi$ 

where  $h_1$  and  $h_2$  are the heights of the waves, measured in millimetres, at any time *t* seconds from the start of the experiment. The two waves join together

The two waves join together.

(i) Express the height  $h_1 + h_2$  of the resultant wave in the form

$$r\cos(t-\alpha)$$
  $0 \le \alpha \le \frac{\pi}{2}$   $r \in \mathbb{R}$  [4]

(ii) Hence find the times when the resultant wave has a height of 3 mm. [4]

4 (i) Given that

 $x^2 + 6xy + y^2 + 32 = 0$ 

use implicit differentiation to show that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{-(x+3y)}{3x+y}$$
[5]

(ii) Hence find the coordinates of the stationary points on the curve

$$x^2 + 6xy + y^2 + 32 = 0$$
 [6]

5 Solve the differential equation

$$\cos^2 4x \ \frac{dy}{dx} = y$$

$$\frac{\pi}{16}$$
[8]

given that  $y = e^3$  when  $x = \frac{\pi}{16}$ 

### 6 (i) Find the vector equation of the line *l* through the points (2, 4, 3) and (1, 2, 3). [4]

The point P lies on the line $l$ . O is the point $(0, 0, 0)$ .	
(ii) Hence, using (i), write down the position vector $\overrightarrow{OP}$	[1]
OP is perpendicular to the line <i>l</i> .	
(iii) Find the coordinates of the point P.	[6]

7 Find the **exact** value of

$$\int_{1}^{3} \ln x \, \mathrm{d}x$$
 [7]

8 (i) Prove the identity

$$\tan 3\theta \equiv \frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}$$
<sup>[7]</sup>

(ii) Hence solve the equation

$$\tan 3\theta = \tan \theta \qquad 0^\circ \le \theta \le 360^\circ$$
<sup>[5]</sup>

## THIS IS THE END OF THE QUESTION PAPER