Rewarding Learning

ADVANCED
General Certificate of Education
2012

Mathematics

Assessment Unit C3

assessing

Module C3: Core Mathematics 3

[AMC31]

AMC31

FRIDAY 18 MAY, MORNING

TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.
Answer **all eight** questions.
Show clearly the full development of your answers.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

Answer all eight questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

1 **(a)** Solve

$$|2x + 3| = 7$$

[4]

(b) Simplify the expression

$$\frac{x^2 + 4x - 21}{x^2 - 25} \div \frac{x + 7}{x - 5}$$

writing it in the form $\dfrac{x + a}{x + b}$, where a and b are integers. [5]

2 **(a)** Express $\dfrac{x - 26}{(x + 2)(x - 5)}$ in partial fractions. [6]

(b) Find the first 3 non-zero terms in the binomial expansion of

$$\frac{1}{1 - 3x}$$

[5]

3 A curve is described by the parametric equations

$$x = 2t - 1 \qquad y = 6 - \tfrac{1}{t}$$

(i) Find the Cartesian equation of this curve. [4]

(ii) Find the point at which this curve crosses the x-axis. [2]

4 **(a)** The cross-section of an earring can be modelled by the area between the curve

$$y = 1 - \frac{1}{x},$$

the line $x = 3$ and the x-axis as shown in **Fig. 1** below.

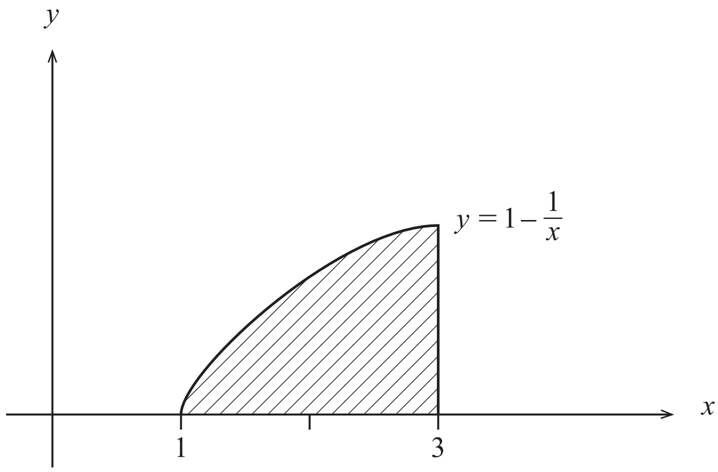

Fig. 1

Find the **exact** cross-sectional area of the earring. [6]

(b) Find

$$\int 5x - \operatorname{cosec}^2 x \, \mathrm{d}x$$

[3]

(c) Find

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\tan 2x}{x-3}\right)$$

[5]

5 **Fig. 2** below shows the graph of the function $y = \text{f}(x)$.

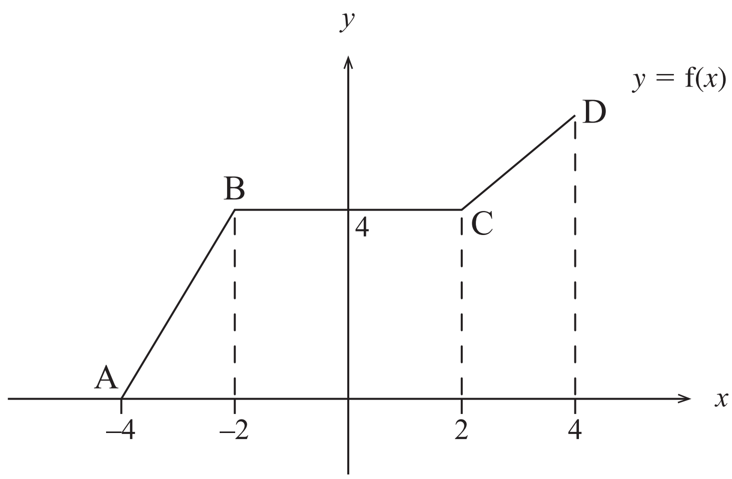

Fig. 2

Fig. 3 below shows the graph of the function $y = a\text{f}(bx)$.

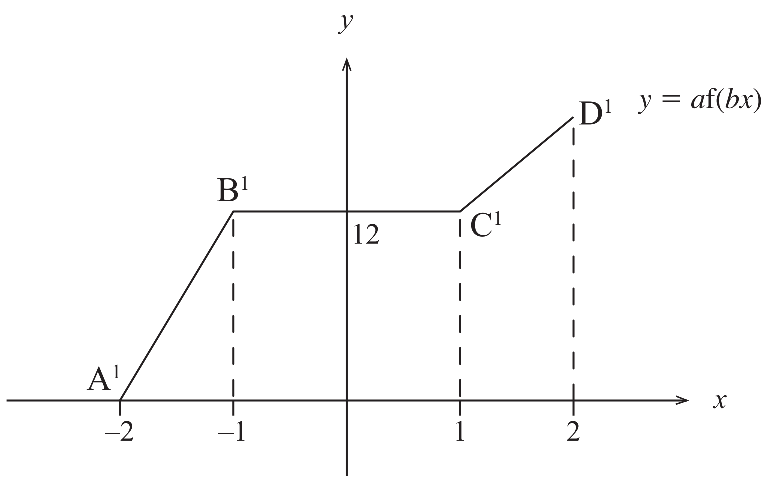

Fig. 3

(i) Write down the values of a and b. [2]

(ii) Sketch the graph of the function $y = -\text{f}(x-2)$, clearly showing the images of A, B, C and D. [3]

6 **(i)** Show that the equation

$$e^{-x} - \sin x = 0$$

has a solution between $x = 0$ and $x = \frac{\pi}{2}$ [3]

(ii) By taking $x = 0$ as a first approximation to this solution and using the Newton-Raphson method twice, find a better approximation. [5]

7 Find the equation of the normal to the curve

$$y = x\sqrt{1+3x} - \ln(3x-2)$$

at the point on the curve where $x = 1$,
giving your answer in the form $ax + by + c = 0$ where a, b and c are integers. [10]

8 **(a)** Prove that

$$(\operatorname{cosec}^2 x - 1)(\sec^2 x - 1) \equiv 1$$

[4]

(b) Solve the equation

$$\sec\left(2x - \frac{\pi}{4}\right) = -2$$

for $0 < x < 2\pi$ [8]

THIS IS THE END OF THE QUESTION PAPER