Rewarding Learning

ADVANCED SUBSIDIARY (AS)
General Certificate of Education
January 2012

## Mathematics

Assessment Unit S1
assessing
Module S1: Statistics 1
[AMS11]


WEDNESDAY 25 JANUARY, AFTERNOON

## TIME

1 hour 30 minutes.

## INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.
Answer all seven questions.
Show clearly the full development of your answers.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or scientific calculator in this paper.

## INFORMATION FOR CANDIDATES

The total mark for this paper is 75
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
A copy of the Mathematical Formulae and Tables booklet is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log _{\mathrm{e}} z$

## Answer all seven questions.

## Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

1 Paul is looking to find the mean and standard deviation of the heights of seventeen-year-olds. He takes a sample within his school and the results are given in Table 1 below.

## Table 1

| Height <br> (to nearest <br> centimetre) | $150-159$ | $160-169$ | $170-179$ | $180-189$ |
| :---: | :---: | :---: | :---: | :---: |
| Frequency | 6 | 29 | 31 | 5 |

Estimate the mean and standard deviation using these data.

2 A continuous random variable $X$ has the probability density function $\mathrm{f}(x)$ defined by

$$
\mathrm{f}(x)=k\left(4 x-x^{3}\right) \quad 0 \leqslant x \leqslant 2
$$

where $k$ is a constant.
(i) Show that $k=\frac{1}{4}$
(ii) Find $\mathrm{P}(X \leqslant 1)$.
(iii) Given that $\mathrm{E}(X)=\frac{16}{15}$, find $\operatorname{Var}(X)$.

3 The number of hits on a website in a one-minute period, $X$, can be modelled as a Poisson distribution with an average rate of $\lambda, \lambda>0$
Write down an expression, in terms of $\lambda$, for:
(i) $\mathrm{P}(X=1)$;
(ii) $\mathrm{P}(X=2)$.

It is known that $\mathrm{P}(X=2)=2 \mathrm{P}(X=1)$.
(iii) Find $\lambda$.
(iv) Hence find the probability of at least three hits on the website in a one-minute period.

4 The time, in seconds, taken by pupils to complete a puzzle is Normally distributed with mean 200 and variance 64
Find the probability that a pupil chosen at random completes the task in:
(i) less than 214 seconds;
(ii) less than 195 seconds;
(iii) less than 195 seconds given that they finish in less than 214 seconds.

Twenty-five percent of pupils complete the task in less than $t$ seconds.
(iv) Find $t$.

5 Sandra is employed by a large supermarket chain to oversee the management of four of its branches in a large town.
Her aim is to increase the yearly profit made by each branch this year.
The supermarket chain reports that nationally the probability of branches increasing their yearly profit is 0.38
(i) What important assumption has to be made in order to model this situation using the Binomial distribution?

Find the probability that there is an increase in this year's profits for:
(ii) all four of Sandra's branches;
(iii) fewer than two of Sandra's branches.

Sandra is paid a $£ 2000$ bonus if all four of her branches increase their profits this year and a $£ 1000$ bonus if either two or three branches increase their profits this year.
(iv) Find Sandra's expected bonus this year.

6 (a) Katy opens a bag of fruit-flavoured sweets which contains 7 strawberry flavoured and 5 lemon flavoured.
She lets her two friends each choose a sweet at random, and then takes one at random herself.
Using a tree diagram, or otherwise, find the probability that Katy chooses a lemon flavoured sweet.
(b) $A$ and $B$ are exhaustive events such that $\mathrm{P}(A)=0.65$ and $\mathrm{P}(B)=0.42$ Find $\mathrm{P}(A \mid B)$.

7 The probability distribution of a discrete random variable, $X$, is given in Table 2 below.

## Table 2

| $x$ | 0 | 1 | 2 | 3 |
| :---: | :---: | :---: | :---: | :---: |
| $\mathrm{P}(X=x)$ | $a$ | 0.4 | 0.2 | $0.4-a$ |

where $a$ is a constant.
(i) Explain briefly why the value of $a$ lies between 0 and 0.4

Find expressions in terms of $a$ for:
(ii) $\mathrm{E}(X)$;
(iii) $\mathrm{E}\left(X^{2}\right)$;
(iv) $\operatorname{Var}(X)$.
(v) Given that $\operatorname{Var}(X)=0.75$, find $a$.

## THIS IS THE END OF THE QUESTION PAPER

