Rewarding Learning

ADVANCED SUBSIDIARY (AS)
General Certificate of Education
2011

Mathematics

Assessment Unit F1

assessing

Module FP1: Further Pure Mathematics 1

[AMF11]

AMF11

FRIDAY 24 JUNE, AFTERNOON

TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.
Answer **all six** questions.
Show clearly the full development of your answers.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

Answer all six questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

1 A circle has equation

$$x^2 + y^2 - 8x - 14y + 40 = 0$$

Find the equation of the tangent to this circle at the point (8, 4) [6]

2 The transformation represented by the matrix **M** maps the points (3, 4) and (5, –2) onto (10, 4) and (8, –2) respectively.

(i) Find the matrix **M**. [4]

The matrix $\mathbf{N} = \begin{pmatrix} -1 & 1 \\ 2 & 0 \end{pmatrix}$

The matrix **S** represents the combined effect of the transformation represented by **N** followed by the transformation represented by **M**.

(ii) Show that $\mathbf{S} = \begin{pmatrix} 0 & 2 \\ 2 & 0 \end{pmatrix}$ [3]

(iii) Find the equations of the straight lines through the origin which are invariant under the transformation represented by **S**. [6]

3 Let **S** be the set of matrices $\begin{pmatrix} p & q \\ 3q & -p \end{pmatrix}$, where p, q are any real numbers.

Prove that **S** forms a group under the operation of matrix addition.

(You may assume that matrix addition is associative.) [9]

4 The matrix **A** is given by $\mathbf{A} = \begin{pmatrix} -1 & p & 0 \\ p & 0 & 2 \\ 0 & 2 & 1 \end{pmatrix}$

One eigenvalue of **A** is 3

(i) Prove that $p = \pm 2$ [6]

Assuming that $p = 2$, find:

(ii) the other eigenvalues of **A** [6]

(iii) an eigenvector corresponding to the eigenvalue 3 [4]

5 A system of equations is given by

$$\begin{aligned} 3x + \lambda y - z &= 3 \\ 2\lambda x + y \quad &= 1 \\ x - y + z \quad &= -2 \end{aligned}$$

(i) Find both values of λ for which this system does not have a unique solution. [6]

(ii) For each of these values of λ decide whether solutions exist and, if they do, find the general solution. [7]

6 **(a)** The complex number z is such that

$$|z| = 4, \ \arg z = \frac{2\pi}{3}$$

Express z in the form $a + b\mathrm{i}$, where a and b are real numbers. [4]

(b) Simplify the number

$$\frac{3-4\mathrm{i}}{2+\mathrm{i}}$$

giving your answer in the form $a + b\mathrm{i}$, where a and b are rational numbers. [5]

(c) **(i)** Sketch on an Argand diagram the locus of those points w which satisfy

$$|w - 3 - 2\mathrm{i}| = 2$$ [3]

(ii) On the same Argand diagram, sketch the locus of those points w which satisfy

$$|w - 3 - 2\mathrm{i}| = |w + 1 + 2\mathrm{i}|$$ [3]

(iii) Show that no point lies on both loci.

A solution by scale drawing will not be accepted. [3]

THIS IS THE END OF THE QUESTION PAPER