Rewarding Learning

ADVANCED
General Certificate of Education
2011

Mathematics

Assessment Unit C4

assessing

Module C4: Core Mathematics 4

[AMC41]

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WEDNESDAY 1 JUNE, MORNING

TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.
Answer **all eight** questions.
Show clearly the full development of your answers.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

Answer all eight questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

1 Given that the points A and B have position vectors:

$$\overrightarrow{OA} = 3\mathbf{i} - \mathbf{j}$$

$$\text{and } \overrightarrow{OB} = 2\mathbf{i} + 6\mathbf{j}$$

find:

(i) the vector $\overrightarrow{AB}$; [2]

(ii) the magnitude of $\overrightarrow{AB}$; [2]

(iii) $\overrightarrow{OA} \,.\, \overrightarrow{OB}$ [2]

(iv) Hence write down the angle AOB. [1]

2 **(i)** Differentiate implicitly with respect to x

$$x + xy - 12$$ [4]

(ii) Hence find the equation of the tangent to the curve

$$x + xy - 12 = 0$$

at the point (2, 5). [3]

3 $2\cos x + 4\sin x$ can be written in the form

$$R\cos(x - \alpha)$$

where α is acute and R is real.

(i) Find R and α. [4]

(ii) Hence solve the equation

$$2\cos x + 4\sin x = 3$$

where $0° \leqslant x \leqslant 360°$ [5]

4 The surface of a goldfish bowl can be modelled by part of the curve

$$y = \sqrt{144 - x^2}$$

being rotated through 2π radians about the x-axis as shown in **Fig. 1** below.

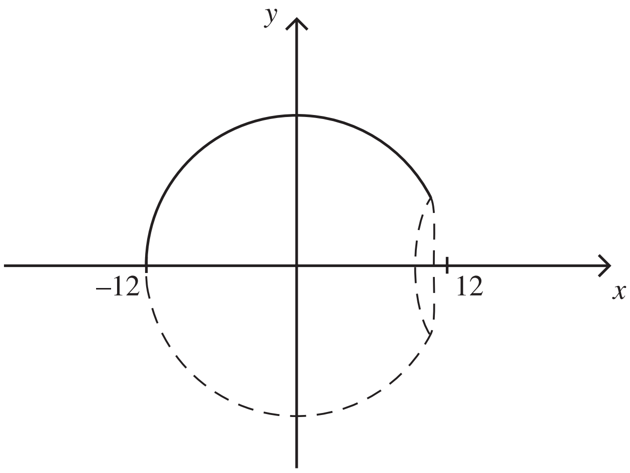

Fig. 1

The radius of the bowl is 12 cm and it is to be filled to a depth of 15 cm.

(i) Find the volume of water in the bowl. [7]

(ii) State one criticism of the model. [1]

5 **(i)** Starting with the appropriate compound angle formula prove that

$$\sin 2A \equiv 2 \sin A \cos A$$ [3]

(ii) Show that

$$\tan A + \cot A \equiv \frac{2}{\sin 2A}$$ [6]

6 The amount x of a substance present in a certain chemical reaction after time t can be modelled by the differential equation

$$\frac{dx}{dt} = k(3 - x)(4 - x)$$

where k is a constant and $x = 0$ when $t = 0$

Given that $x = 2$ when $t = 10$, find the value of k. [14]

7 **Fig. 2** below shows a sketch of the graph whose equation is

$$y = \frac{2x + 3}{x - 1}$$

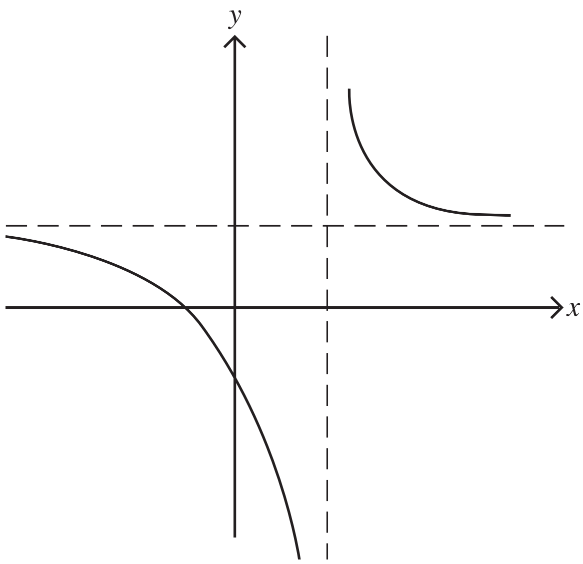

Fig. 2

(i) Write down the equations of the asymptotes to this graph. [2]

The function f, with domain $x > 1$, is defined by

$$\mathrm{f}: x \rightarrow \frac{2x + 3}{x - 1}$$

(ii) Find the inverse function f^{-1}, stating its domain. [6]

8 **(i)** Using integration by parts, show that

$$\int x \ln x \, dx = \frac{x^2}{2} \ln x - \frac{x^2}{4} + c$$ [6]

(ii) Using **(i)**, find

(a) $\int x \ln x^2 \, dx$ [3]

(b) $\int x \ln 3x \, dx$ [4]

THIS IS THE END OF THE QUESTION PAPER

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