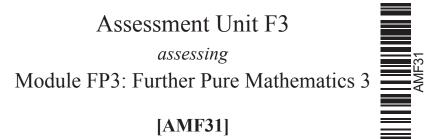


ADVANCED General Certificate of Education 2011

Mathematics



THURSDAY 26 MAY, MORNING

TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided. Answer **all seven** questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75

Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.

A copy of the Mathematical Formulae and Tables booklet is provided.

Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$



6222

Answer all seven questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

1 (i) If $x = 5 \cos \theta - 3$ show that

$$16 - 6x - x^2 = 25\sin^2\theta$$
 [3]

(ii) Hence or otherwise show that

$$\int \frac{1}{\sqrt{16 - 6x - x^2}} \, \mathrm{d}x = -\cos^{-1} \left(\frac{x+3}{5}\right) + c \tag{4}$$

2 (i) Using the exponential definitions of $\cosh x$ and $\sinh x$ prove that

$$2\cosh 4x \cosh x \equiv \cosh 5x + \cosh 3x$$
[3]

[4]

(ii) Hence solve, for real values of x, the equation

$$\cosh 5x + \cosh 3x = 4 \cosh x$$

leaving your answers in logarithmic form.

3 (i) Sketch the curve $y = \sinh^{-1} x$

(ii) Show that

$$\frac{d}{dx}(\sinh^{-1}x) = \frac{1}{\sqrt{1+x^2}}$$
 [4]

(iii) Show that

$$\sinh^{-1} x \equiv \ln\left(x + \sqrt{1 + x^2}\right)$$
[4]

(iv) Find, in fraction form, the exact solution to the equation

$$\sinh^{-1}\frac{3}{4} + \sinh^{-1} x = \sinh^{-1}\frac{4}{3}$$
 [4]

4 A plane Π has vector equation

$$\mathbf{r.}(2\mathbf{i}-3\mathbf{j}+\mathbf{k})=5$$

(i) Find the shortest distance from the origin to the plane. [2]

The line L has Cartesian equation

$$\frac{x-2}{3} = \frac{y-5}{-2} = \frac{z+1}{5}$$

- (ii) Find the coordinates of the point where the line L meets the plane Π [4]
- (iii) Find the angle between the line L and the plane Π [5]

5 (i) Given that

$$I_n = \int_0^{\frac{\pi}{2}} x^n \cos x \, \mathrm{d}x$$

show that for $n \ge 2$

$$I_{n} = \left(\frac{\pi}{2}\right)^{n} - n(n-1)I_{n-2}$$
 [6]

(ii) Hence evaluate, in terms of π ,

$$\int_0^{\frac{\pi}{2}} x^4 \cos x \, \mathrm{d}x \tag{5}$$

6 (a) (i) Show that if

$$y = \sin^{-1}\sqrt{x} - \sqrt{x(1-x)} \qquad |x| < |$$

then $\frac{dy}{dx} = \sqrt{\frac{x}{1-x}}$ [5]

(ii) Hence or otherwise find the exact value of

$$\int_{0}^{\frac{1}{2}} \sqrt{\frac{x}{1-x}} \, \mathrm{d}x$$
 [2]

(b) Show that

$$\int_{0}^{2} \frac{4 - 3x}{4 + 3x^{2}} \, \mathrm{d}x = \frac{2\pi}{3\sqrt{3}} - \ln 2 \tag{6}$$

7 With reference to a fixed origin O the points A(4, 1, 3), B(-2, 7, 6) and C(5, -3, 2) determine the plane ABC.

(i) Find
$$\overrightarrow{AB} \times \overrightarrow{AC}$$
 [4]

(ii) Hence or otherwise find, in the form $\mathbf{r.n} = d$, an equation of the plane ABC. [3]

The point D with position vector $\overrightarrow{OD} = 11\mathbf{i} - 9\mathbf{j} + \lambda\mathbf{k}$ is in the plane ABC.

THIS IS THE END OF THE QUESTION PAPER