Rewarding Learning

ADVANCED
General Certificate of Education
2011

Mathematics

Assessment Unit C3

assessing

Module C3: Core Mathematics 3

AMC31

[AMC31]

FRIDAY 20 MAY, AFTERNOON

TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.
Answer **all eight** questions.
Show clearly the full development of your answers.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

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Answer all eight questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

1 Find the first 3 terms in the binomial expansion of $\sqrt{1+2x}$ [5]

2 **Fig. 1** below shows the graphs of

$$y = |3 - 2x| \quad \text{and} \quad y = 2$$

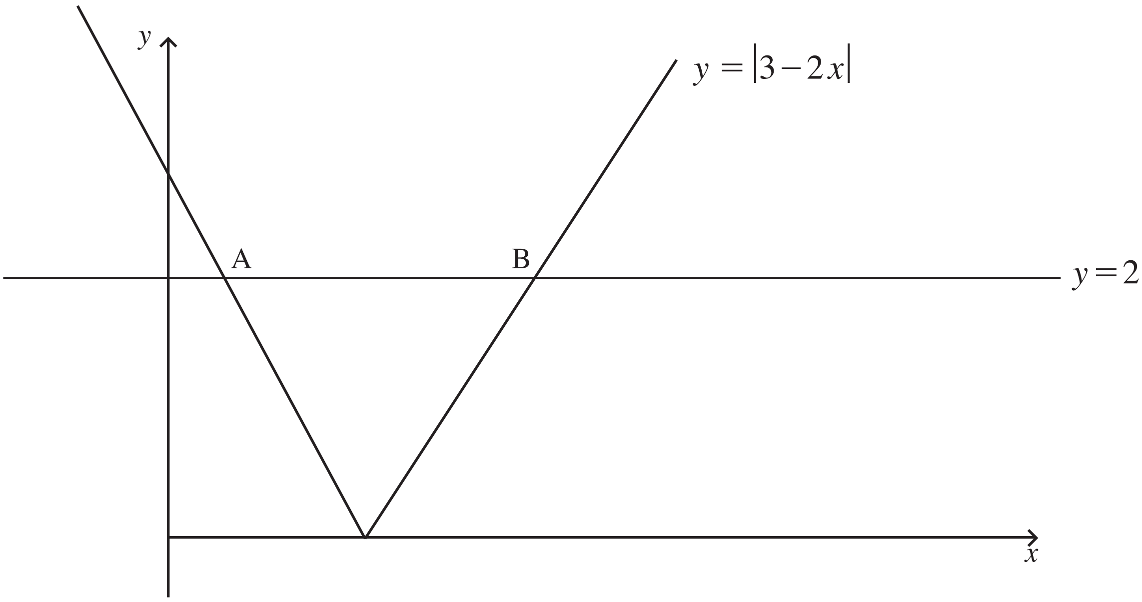

Fig. 1

The graphs intersect at the points A and B.

Find the x coordinates of A and B. [6]

3 Use partial fractions to rewrite

$$\frac{x^2 + 8x - 1}{(x-3)(x-1)^2}$$

in the form

$$\frac{A}{x-3} + \frac{B}{x-1} + \frac{C}{(x-1)^2}$$

where A, B and C are integers. [5]

4 A population of microorganisms grows according to the rule

$$N = 15000\,\mathrm{e}^{0.7t}$$

where N is the size of the population at time t hours.

(i) Find the initial population. [1]

(ii) Find how long it will take for the population to treble. [6]

5 **Fig. 2** below shows the graphs of

$$y = \sin x \quad \text{and} \quad y = 1 - x^3$$

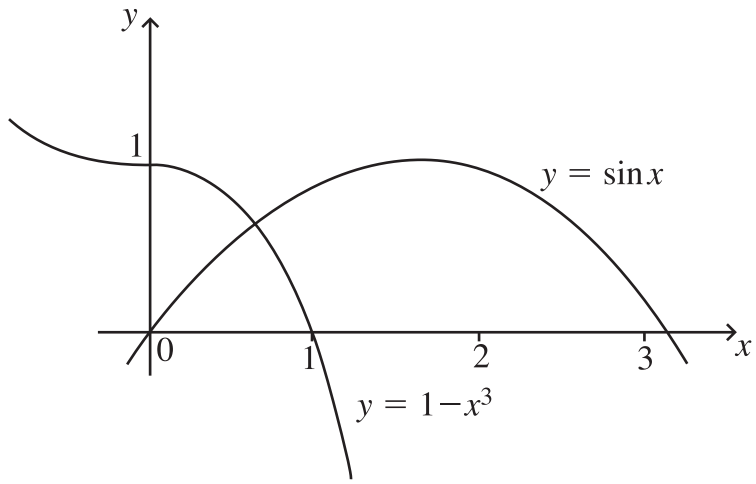

Fig. 2

(i) Show that the point of intersection of these graphs can be found by solving the equation

$$\sin x + x^3 - 1 = 0$$ [2]

(ii) Verify that this value of x lies between $x = 0$ and $x = 1$ [3]

(iii) Taking $x = 0.5$ as a first approximation to this value of x, use the Newton-Raphson method twice to find a better approximation. [5]

6 **(a)** Find

$$\int x^3 - \frac{2}{x} + \operatorname{cosec}^2 x - e^{-3x}\, dx$$

[5]

(b) A component of a machine is to be cut from flat steel. It can be modelled as the area between the curve $y = \cos 2x$, the axes and the line $x = \dfrac{\pi}{6}$
This is shown shaded in **Fig. 3** below.

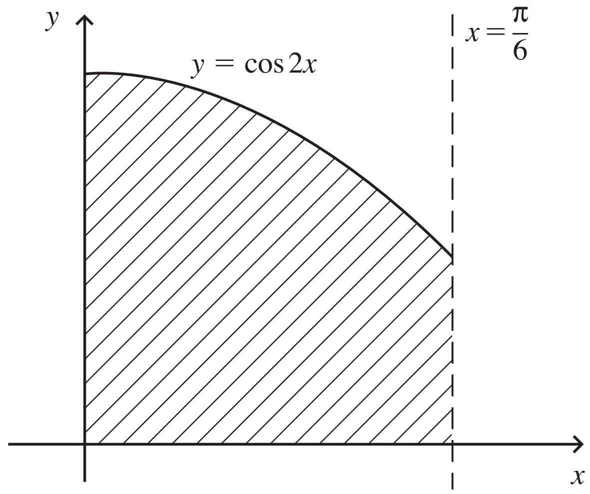

Fig. 3

Using calculus, determine the area of the component. [6]

7 **(a)** Differentiate

$$\frac{x^4}{\tan^2 x}$$

[6]

(b) Find the **exact** equation of the tangent to the curve

$$y = x \ln x$$

at the point where $x = 2$ [8]

8 **(a)** A circle is defined by the parametric equations

$$x = -1 + 3\sin\theta \qquad\qquad y = 2 + 3\cos\theta$$

(i) Find the cartesian equation of this circle. [4]

(ii) Write down the centre and radius of this circle. [3]

(b) **(i)** Prove the identity

$$\frac{1-\sin\theta}{1+\cos\theta} \times \frac{1+\sin\theta}{1-\cos\theta} \equiv \cot^2\theta$$ [4]

(ii) Hence solve the equation

$$\frac{1-\sin\theta}{1+\cos\theta} \times \frac{1+\sin\theta}{1-\cos\theta} = \cot\theta + 2$$

where $-\pi \leqslant \theta \leqslant \pi$ [6]

THIS IS THE END OF THE QUESTION PAPER

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