Rewarding Learning

ADVANCED
General Certificate of Education
January 2011

Mathematics

Assessment Unit F1

assessing

Module FP1: Further Mathematics 1

[AMF11]

WEDNESDAY 19 JANUARY, AFTERNOON

MARK SCHEME

GCE ADVANCED/ADVANCED SUBSIDIARY (AS) MATHEMATICS

Introduction

The mark scheme normally provides the most popular solution to each question. Other solutions given by candidates are evaluated and credit given as appropriate; these alternative methods are not usually illustrated in the published mark scheme.

The marks awarded for each question are shown in the right hand column and they are prefixed by the letters **M**, **W** and **MW** as appropriate. The key to the mark scheme is given below:

M indicates marks for correct method.

W indicates marks for working.

MW indicates marks for combined method and working.

The solution to a question gains marks for correct method and marks for an accurate working based on this method. Where the method is not correct no marks can be given.

A later part of a question may require a candidate to use an answer obtained from an earlier part of the same question. A candidate who gets the wrong answer to the earlier part and goes on to the later part is naturally unaware that the wrong data is being used and is actually undertaking the solution of a parallel problem from the point at which the error occurred. If such a candidate continues to apply correct method, then the candidate's individual working must be followed through from the error. If no further errors are made, then the candidate is penalised only for the initial error. Solutions containing two or more working or transcription errors are treated in the same way. This process is usually referred to as "follow-through marking" and allows a candidate to gain credit for that part of a solution which follows a working or transcription error.

Positive marking:

It is our intention to reward candidates for any demonstration of relevant knowledge, skills or understanding. For this reason we adopt a policy of **following through** their answers, that is, having penalised a candidate for an error, we mark the succeeding parts of the question using the candidate's value or answers and award marks accordingly.

Some common examples of this occur in the following cases:

(a) a numerical error in one entry in a table of values might lead to several answers being incorrect, but these might not be essentially separate errors;

(b) readings taken from candidates' inaccurate graphs may not agree with the answers expected but might be consistent with the graphs drawn.

When the candidate misreads a question in such a way as to make the question easier only a proportion of the marks will be available (based on the professional judgement of the examining team).

			AVAILABLE MARKS
1 **(i)**	Det $\lvert \mathbf{A} - \lambda \mathbf{I} \rvert = 0$	M1	
	Hence		
	$\Rightarrow \begin{vmatrix} 7-\lambda & -4 \\ -2 & 5-\lambda \end{vmatrix} = 0$	M1	
	$\Rightarrow (7-\lambda)(5-\lambda) - 8 = 0$	W1	
	$\Rightarrow 35 - 12\lambda + \lambda^2 - 8 = 0$		
	$\Rightarrow \lambda^2 - 12\lambda + 27 = 0$	W1	
	$\Rightarrow (\lambda - 9)(\lambda - 3) = 0$		
	$\Rightarrow \lambda = 3, 9$	W1	
(ii)	$\begin{pmatrix} 7 & -4 \\ -2 & 5 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 9 \begin{pmatrix} x \\ y \end{pmatrix}$	M1	
	Hence $7x - 4y = 9x \Rightarrow -4y = 2x$	W1	
	$-2x + 5y = 9y \Rightarrow -2x = 4y$		
	Therefore, an eigenvector is $\begin{pmatrix} -2 \\ 1 \end{pmatrix}$	W1	
	The corresponding unit eigenvector is then $\frac{1}{\sqrt{5}} \begin{pmatrix} -2 \\ 1 \end{pmatrix}$	W1	9

AVAILABLE MARKS

2 **(i)** $x^2 + y^2 + 2x - 6y + 8 = 0$ ①

$x^2 + y^2 - 4x - 28 = 0$ ②

Subtract to give $6x - 6y + 36 = 0$ M1W1

Re-arrange to give $x = y - 6$

Substitute into equation ② M1

$\Rightarrow (y - 6)^2 + y^2 - 4(y - 6) - 28 = 0$ W1

Expand to give

$\Rightarrow y^2 - 12y + 36 + y^2 - 4y + 24 - 28 = 0$ W1

$\Rightarrow 2y^2 - 16y + 32 = 0$

$\Rightarrow y^2 - 8y + 16 = 0$ W1

$\Rightarrow (y - 4)^2 = 0$

Hence $y = 4$ W1

Therefore $x = -2$ W1

The point of intersection is then (–2, 4)

(ii) Circle 1 has centre $C_1(-1, 3)$ MW1

Circle 2 has centre $C_2(2, 0)$ MW1

Since the point of intersection P(–2, 4) does not lie between MW1
C_1 and C_2 then the circles touch internally. MW1 12

AVAILABLE MARKS

3 **(a)** Choice of any two suitable values such as 2 and 4 M1

Multiplying these values gives $2 \times 4 = 0$ W1

Since 0 is not in the set, then closure does not hold and a group is not formed. W1

(b) **(i)**

	0	1	2	3	4	5	6	7
0	0	1	2	3	4	5	6	7
1	1	2	3	4	5	6	7	0
2	2	3	4	5	6	7	0	1
3	3	4	5	6	7	0	1	2
4	4	5	6	7	0	1	2	3
5	5	6	7	0	1	2	3	4
6	6	7	0	1	2	3	4	5
7	7	0	1	2	3	4	5	6

MW5

(ii) If $x^3 = x,$

then $x = 0, 4$ MW2

(ii) $\{0, 2, 4, 6\}$ MW3 13

			AVAILABLE MARKS
4	**(a)** The matrix represents a reflection, with	MW1	
	$\cos 2\theta = -\frac{3}{5}$ and	MW1	
	$\sin 2\theta = \frac{4}{5}$		
	Hence $\tan 2\theta = -\frac{4}{3}$ and 2θ must lie in the 2nd quadrant	W1	
	Therefore, $2\theta = 126.9°$		
	$\Rightarrow \tan\theta = 2$	W1	
	Therefore the matrix represents a reflection in the line $y = 2x$	MW1	
	(b)		
	$\begin{pmatrix} 3 & -2 \\ 2 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} X \\ Y \end{pmatrix}$	M1	
	$\Rightarrow \begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{1}\begin{pmatrix} -1 & 2 \\ -2 & 3 \end{pmatrix}\begin{pmatrix} X \\ Y \end{pmatrix}$	M1W1	
	$\Rightarrow x = -X + 2Y$		
	and $\Rightarrow y = -2X + 3Y$	W1	
	Substitute into $x^2 + y^2 = 25$	M1	
	$\Rightarrow (-X + 2Y)^2 + (-2X + 3Y)^2 = 25$	W1	
	Expand to give		
	$X^2 - 4XY + 4Y^2 + 4X^2 - 12XY + 9Y^2 = 25$	W1	
	$\Rightarrow 5X^2 + 13Y^2 - 16XY = 25$	W1	13

AVAILABLE MARKS

5

(i) Determinant is given by $\begin{vmatrix} 3 & 2 & a \\ 1 & -2 & -1 \\ a & 0 & 3 \end{vmatrix}$

Expand to give $3(-6) - 2(3 + a) + a(2a)$ M1

$= -18 - 6 - 2a + 2a^2$ W1

Hence determinant of **M** is $2a^2 - 2a - 24$ W1

(ii) For a unique solution then $\begin{vmatrix} 3 & 2 & a \\ 1 & -2 & -1 \\ a & 0 & 3 \end{vmatrix} \neq 0$

Hence $2a^2 - 2a - 24 \neq 0$ M1

$\Rightarrow a^2 - a - 12 \neq 0$

$\Rightarrow (a - 4)(a + 3) \neq 0$

Therefore, if the solution is unique then $a \neq 4$, $a \neq -3$ W2

(iii) If $a = 1$, then det $\mathbf{M} = 2(1)^2 - 2(1) - 24$

$= -24$ MW1

Matrix of minors = $\begin{pmatrix} -6 & 4 & 2 \\ 6 & 8 & -2 \\ 0 & -4 & -8 \end{pmatrix}$ MW2

Matrix of cofactors = $\begin{pmatrix} -6 & -4 & 2 \\ -6 & 8 & 2 \\ 0 & 4 & -8 \end{pmatrix}$ MW1

Transpose = $\begin{pmatrix} -6 & -6 & 0 \\ -4 & 8 & 4 \\ 2 & 2 & -8 \end{pmatrix}$ MW1

Inverse = $-\dfrac{1}{24}\begin{pmatrix} -6 & -6 & 0 \\ -4 & 8 & 4 \\ 2 & 2 & -8 \end{pmatrix}$ MW1

(iv) Solution to the equations = $-\dfrac{1}{24}\begin{pmatrix} -6 & -6 & 0 \\ -4 & 8 & 4 \\ 2 & 2 & -8 \end{pmatrix}\begin{pmatrix} 7 \\ 1 \\ 11 \end{pmatrix}$ M1

$= -\dfrac{1}{24}\begin{pmatrix} -48 \\ 24 \\ -72 \end{pmatrix}$ W1

Hence $x = 2$, $y = -1$, $z = 3$ W1 15

AVAILABLE MARKS

6 **(a)** $z = \dfrac{2i \pm \sqrt{-4 + 40}}{4}$ M1W1

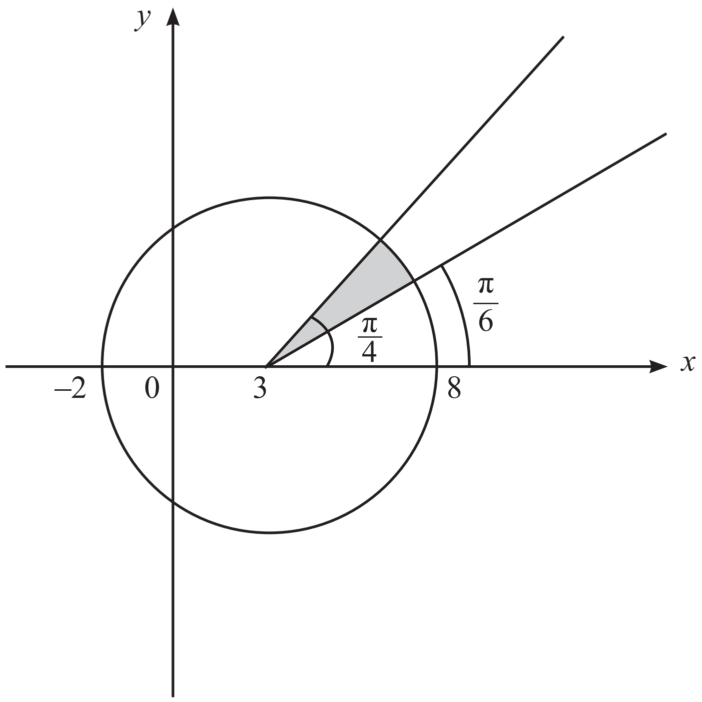
$\Rightarrow z = \dfrac{2i \pm 6}{4}$ W1

$\Rightarrow z = \dfrac{1}{2} i \pm \dfrac{3}{2}$ W1

(b) **(i)** Circle, centre (3,0) and with radius 5 MW3

See diagram in part (ii)

(ii)



Half line through (3, 0) at angle of $\dfrac{\pi}{6}$ to x – axis MW3

Half line through (3, 0) at angle of $\dfrac{\pi}{4}$ to x – axis MW1

Shading between lines MW1

Shading inside circle MW1 13

Total **75**