Rewarding Learning

ADVANCED<br>General Certificate of Education<br>January 2011

Mathematics


FRIDAY 14 JANUARY, AFTERNOON

## TIME

1 hour 30 minutes.

## INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.
Answer all seven questions.
Show clearly the full development of your answers.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or scientific calculator in this paper.

## INFORMATION FOR CANDIDATES

The total mark for this paper is 75
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
A copy of the Mathematical Formulae and Tables booklet is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log _{\mathrm{e}} z$

## Answer all seven questions.

## Show clearly the full development of your answers.

## Answers should be given to three significant figures unless otherwise stated.

1 Simplify

$$
\begin{equation*}
\frac{x^{2}+x-6}{x^{2}-9} \div \frac{x-2}{4} \tag{5}
\end{equation*}
$$

2 A curve is defined by the parametric equations

$$
x=\sin \theta+1 \quad y=2 \cos \theta-1
$$

(i) Find the cartesian equation of this curve.
(ii) Find the points where this curve crosses the $x$-axis.

3 Fig. 1 below shows the graph of $y=\ln x$


Fig. 1
(i) Sketch the graph of

$$
y=\ln (x+2)
$$

showing the vertical asymptote.
Write down the coordinates of $\mathrm{A}^{\prime}$, the image of the point A .
(ii) Sketch the graph of

$$
\begin{equation*}
y=|\ln (x+2)| \tag{2}
\end{equation*}
$$

(iii) Find the exact values of $x$ for which

$$
\begin{equation*}
|\ln (x+2)|=2 \tag{5}
\end{equation*}
$$

4 Solve the equation

$$
\begin{equation*}
\tan ^{2} \theta=1-\sec \theta \tag{8}
\end{equation*}
$$

where $0 \leqslant \theta \leqslant 2 \pi$

5 Differentiate with respect to $x$
(i) $\frac{x^{2}}{\ln x}$
(ii) $x \sec x$
(iii) $\cot ^{3}(2 x)$

6 The cross-section through a half-pipe in a skate park can be modelled by the curve

$$
y=\frac{4}{x+1}+\frac{\mathrm{e}^{x}}{5}
$$

between $x=0$ and $x=3$ metres as shown in Fig. 2 below:


Fig. 2
(i) By using Simpson's Rule with 6 strips, find an approximate value for the shaded area. [6]
(ii) Find the exact value of the shaded area.

7 (i) By using partial fractions, show that

$$
\begin{equation*}
\frac{2-x}{(1+2 x)(3+x)}=\frac{1}{1+2 x}-\frac{1}{3+x} \tag{7}
\end{equation*}
$$

(ii) Hence, using the binomial theorem, expand,

$$
\frac{2-x}{(1+2 x)(3+x)}
$$

in ascending powers of $x$, up to and including the term in $x^{2}$
(iii) Find the range of values of $x$ for which the expression is valid.

## THIS IS THE END OF THE QUESTION PAPER

