



Rewarding Learning

ADVANCED
General Certificate of Education
2010

Mathematics

Assessment Unit M3

assessing

Module M3: Mechanics 3

[AMM31]



TUESDAY 15 JUNE, MORNING

TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.

Answer **all six** questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75

Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.

Answers should include diagrams where appropriate and marks may be awarded for them.

Take $g = 9.8 \text{ ms}^{-2}$, unless specified otherwise.

A copy of the **Mathematical Formulae and Tables booklet** is provided.

Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that

$\ln z \equiv \log_e z$

Answer all six questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

- 1 An isosceles triangular lamina ABC of uniform density has sides $AB = BC = 30$ cm and $AC = 48$ cm. D, E and F are the midpoints of AC, AB and BC respectively and are joined to make four congruent triangles, each similar to ABC. H is the midpoint of EF as shown in Fig. 1 below.

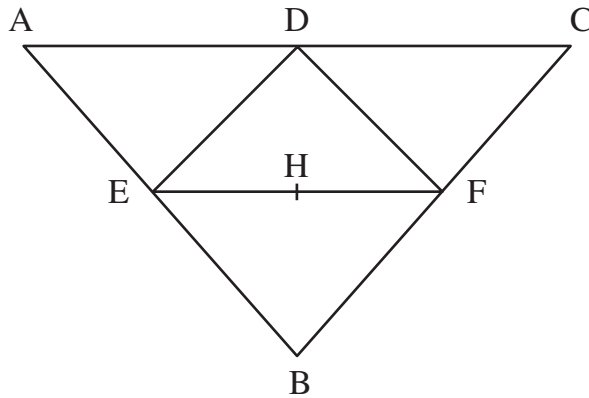


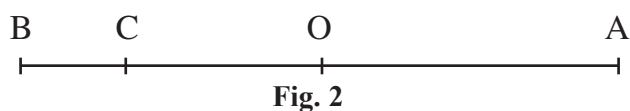
Fig. 1

- (i) Find the lengths of BD and BH. [3]

The triangular lamina EBF is removed. The centre of mass of the remaining trapezoidal lamina is on HD, d cm from H.

- (ii) Find d . [5]

- 2 A particle P moves with S.H.M. between the points A and B on a straight horizontal line as shown in **Fig. 2** below.



O is the centre of the motion.

AB is 2 m.

OC is 0.8 m

At time $t = 0$ seconds, P is at A and is moving towards O.

The maximum acceleration of P is 9 m s^{-2}

- (i) Find the period of the motion. [5]
- (ii) Find the speed of the particle at C. [3]
- (iii) Find the least time taken for the particle to travel from A to C. [5]

- 3 At the Kiwee Cave Bungee Jump Site, jumpers drop on an elastic cord from a horizontal platform that extends over the edge at the top of a cliff. Benji, of mass 60 kg, is attached to a cord of natural length 12 m and modulus of elasticity 240g newtons. The other end of the cord is attached firmly to the platform.

Benji drops from the platform, and when she stops oscillating and hangs in equilibrium, the extension in the cord is e metres.

- (i) Find e . [4]

When jumpers eventually come to rest and hang in equilibrium, they are pulled over into the cave in the cliff face and exit through a tunnel.

The floor of the cave is 15 m below the top of the cliff.

As Benji descends, she passes the cave and comes instantaneously to rest at a point P, d metres below the floor of the cave as shown in Fig. 3 below.

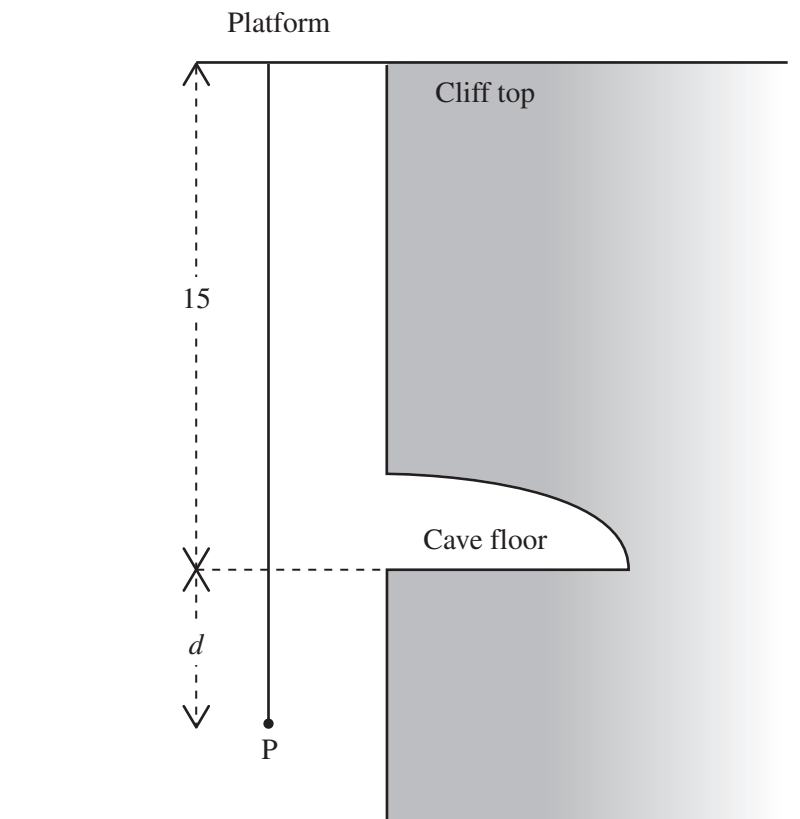


Fig. 3

- (ii) Taking Gravitational Potential Energy to be zero at the level of the floor of the cave, find d . [6]

- (iii) Find the resultant force acting on Benji at P. [3]

- 4 (a) A particle moves x metres from a fixed origin along the x -axis under the action of a variable force F newtons where

$$F = 3(x - 2)^2$$

- (i) Show that W joules, the work done by F , when the particle has moved d metres, can be expressed as

$$W = (d - 2)^3 + 8 \quad [4]$$

- (ii) If $W = 35$, find d . [2]

- (b) When a particle is acted on by a set of forces, it is displaced by the vector \mathbf{s} metres where

$$\mathbf{s} = \begin{pmatrix} a - 1 \\ 1 - 2a \\ a \end{pmatrix}$$

where a is a constant.

One of the forces, \mathbf{F} newtons, is given by

$$\mathbf{F} = 3 \begin{pmatrix} 2 \\ -5 \\ a \end{pmatrix}$$

- (i) Find in terms of a , the work, D joules, done by \mathbf{F} in displacing the particle by \mathbf{s} . [3]
- (ii) If $D = 114$, find a . [3]
- (iii) Write down the values of \mathbf{F} and \mathbf{s} when $a = 3$ [2]
- (iv) When $a = 3$, what can be said about \mathbf{s} and \mathbf{F} and hence what can be said about the set of forces? [2]

- 5 **Fig. 4** below shows a particle P of weight W newtons attached to two light elastic strings. The other ends of the strings are attached to two fixed points A and B. When the particle is in equilibrium, PA is 40° above the horizontal and PB is 20° below the horizontal.

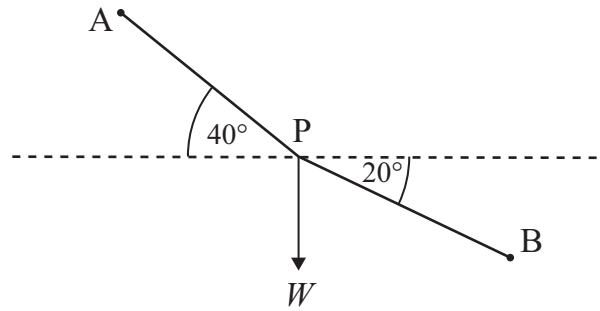


Fig. 4

The string PA has natural length 0.8 m and modulus 184 N.
The tension in PA is 92 N.

- (i) Find the extension in PA. [3]
- (ii) By resolving perpendicular to PB, find W . [4]
- (iii) Show that the tension in the string PB is 75 N. [3]

The string PB has natural length 0.9 m and modulus λ .
The extension in PB is $\frac{1}{4}$ of its extended length.

- (iv) Find λ . [3]

- 6 Faro airport is 2000 km due south of Belfast. Early one morning, a plane flies from Belfast to Faro. The wind blows steadily at 80 km h^{-1} from the west. The plane flies at 700 km h^{-1} in still air. The pilot sets his course on a bearing of $(180 + \theta)^\circ$ as shown in **Fig. 5** below.

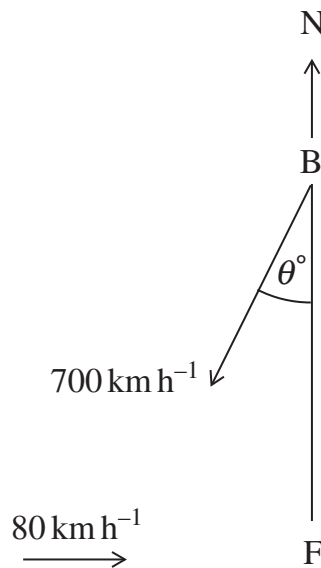


Fig. 5

- (i) Find θ . [2]

Some time later, the plane returns to Belfast. The plane again flies at 700 km h^{-1} in still air. The wind has the same speed, but has changed direction to α° west of north where $\sin \alpha^\circ = 0.8$ as shown in **Fig. 6** below.

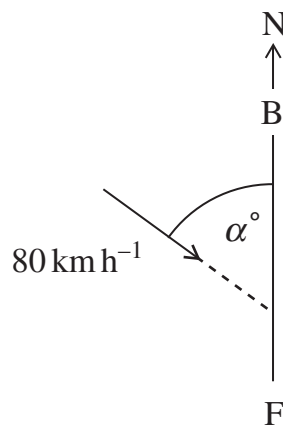


Fig. 6

- (ii) Draw a suitable velocity diagram and find the bearing of the course the pilot should set in order to return to Belfast. [6]
- (iii) Find, in hours, how long the return flight will take. [4]

