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ADVANCED
General Certificate of Education 2010

## Mathematics

# Assessment Unit C3 <br> assessing <br> Module C3: Core Mathematics 3 

[AMC31]
WEDNESDAY 2 JUNE, AFTERNOON

## TIME

1 hour 30 minutes.

## INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.
Answer all eight questions.
Show clearly the full development of your answers.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or a scientific calculator in this paper.

## INFORMATION FOR CANDIDATES

The total mark for this paper is 75
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
A copy of the Mathematical Formulae and Tables booklet is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log _{\mathrm{e}} z$

## Answer all eight questions.

## Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

Solve

$$
\begin{equation*}
|5 x+3|<2 \tag{4}
\end{equation*}
$$

2 (i) Sketch, on the same diagram, the graphs of

$$
\begin{equation*}
y=\sin x \quad \text { and } \quad y=\cos x \quad \text { for } 0^{\circ} \leqslant x \leqslant 360^{\circ} \tag{2}
\end{equation*}
$$

(ii) Given that $\sin x \equiv \cos (x-a)$, write down a possible value of $a$.
(iii) Sketch the graph of

$$
y=\sin (2 x)-1
$$

$$
\begin{equation*}
\text { for } 0^{\circ} \leqslant x \leqslant 360^{\circ} \tag{2}
\end{equation*}
$$

3 (a) Differentiate with respect to $x$

$$
\begin{equation*}
x^{2} \ln x \tag{3}
\end{equation*}
$$

(b) Find

$$
\begin{equation*}
\int\left(3 x^{2}+\mathrm{e}^{-x}-\operatorname{cosec} x \cot x+\frac{3}{x}\right) \mathrm{d} x \tag{5}
\end{equation*}
$$

4 A curve is defined by the parametric equations

$$
x=\tan t-1 \quad y=\cot ^{2} t+1
$$

Find the cartesian equation of this curve.

5 (a) Use partial fractions to rewrite

$$
\begin{equation*}
\frac{2 x-7}{(x-3)^{2}} \tag{6}
\end{equation*}
$$

(b) Find the first 3 terms in the binomial expansion of

$$
\begin{equation*}
\frac{1}{(3-x)^{2}} \tag{7}
\end{equation*}
$$

6 (a) The equation $4 \mathrm{e}^{-x}-x=0$ has a root which is approximately 1.3 Starting with this value for $x$, use the Newton-Raphson method twice to find a better approximation to the root.
(b) The amount of the Carbon-14 isotope remaining in a substance after $t$ years can be written as

$$
N=N_{0} \mathrm{e}^{-k t}
$$

where $N_{0}$ is the amount of the substance when $t=0$
The half-life of the Carbon-14 isotope is 5730 years.
(i) Find the value of the constant $k$.
(ii) Calculate what percentage of the isotope will be left after 1000 years.

7 (a) Solve the equation

$$
\begin{equation*}
4 \sin x+1=3 \operatorname{cosec} x \tag{7}
\end{equation*}
$$

for $0^{\circ} \leqslant x \leqslant 360^{\circ}$
(b) Prove the identity

$$
\begin{equation*}
\operatorname{cosec} 2 \theta-\cot 2 \theta \equiv \tan \theta \tag{6}
\end{equation*}
$$

8 The logo for a ski lift company is shown in Fig. 1 below.


Fig. 1
It can be modelled by a part of the curve $y=\sec ^{2} x$ between $x=-\frac{\pi}{3}$ and $x=\frac{\pi}{3}$
(i) Find the exact area of the shaded region.

PQ is the tangent to the curve at $x=-\frac{\pi}{6}$
(ii) Find the equation of this tangent.

## THIS IS THE END OF THE QUESTION PAPER

