

ADVANCED General Certificate of Education 2010

# **Mathematics**

Assessment Unit C3 assessing Module C3: Core Mathematics 3

## [AMC31]

### WEDNESDAY 2 JUNE, AFTERNOON

#### TIME

1 hour 30 minutes.

#### **INSTRUCTIONS TO CANDIDATES**

Write your Centre Number and Candidate Number on the Answer Booklet provided. Answer **all eight** questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

You are permitted to use a graphic or a scientific calculator in this paper.

#### **INFORMATION FOR CANDIDATES**

The total mark for this paper is 75

Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.

A copy of the Mathematical Formulae and Tables booklet is provided.

Throughout the paper the logarithmic notation used is  $\ln z$  where it is noted that  $\ln z \equiv \log_e z$ 

Answer all eight questions.

#### Show clearly the full development of your answers.

### Answers should be given to three significant figures unless otherwise stated.

1 Solve

$$|5x+3| < 2$$
 [4]

2 (i) Sketch, on the same diagram, the graphs of

$$y = \sin x$$
 and  $y = \cos x$  for  $0^{\circ} \le x \le 360^{\circ}$  [2]

- (ii) Given that  $\sin x \equiv \cos (x a)$ , write down a possible value of a. [1]
- (iii) Sketch the graph of

$$y = \sin(2x) - 1$$
[2]

for  $0^{\circ} \le x \le 360^{\circ}$ 

3 (a) Differentiate with respect to x

$$x^2 \ln x$$
 [3]

(b) Find 
$$\int \left(3x^2 + e^{-x} - \csc x \cot x + \frac{3}{x}\right) dx$$
 [5]

4 A curve is defined by the parametric equations

$$x = \tan t - 1 \qquad y = \cot^2 t + 1$$

Find the cartesian equation of this curve.

[5]

5 (a) Use partial fractions to rewrite

$$\frac{2x-7}{(x-3)^2}$$
 [6]

(b) Find the first 3 terms in the binomial expansion of

$$\frac{1}{(3-x)^2}$$
[7]

- 6 (a) The equation  $4e^{-x} x = 0$  has a root which is approximately 1.3 Starting with this value for x, use the Newton-Raphson method twice to find a better approximation to the root.
  - (b) The amount of the Carbon-14 isotope remaining in a substance after *t* years can be written as

$$N = N_0 e^{-kt}$$

where  $N_0$  is the amount of the substance when t = 0

The half-life of the Carbon-14 isotope is 5730 years.

- (i) Find the value of the constant k. [3]
- (ii) Calculate what percentage of the isotope will be left after 1000 years. [2]
- 7 (a) Solve the equation

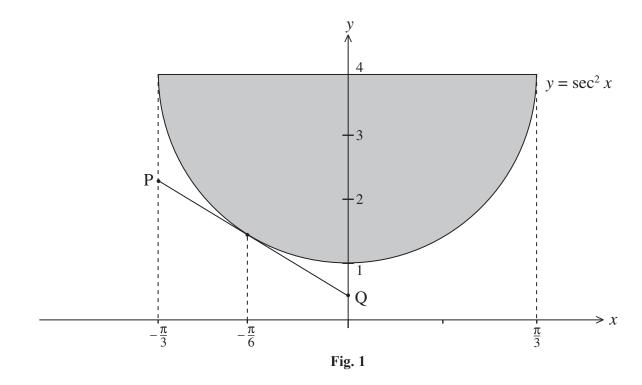
$$4\sin x + 1 = 3\operatorname{cosec} x$$

- for  $0^{\circ} \le x \le 360^{\circ}$  [7]
- (b) Prove the identity

$$\operatorname{cosec} 2\theta - \cot 2\theta \equiv \tan \theta$$
 [6]

[7]

8 The logo for a ski lift company is shown in **Fig. 1** below.



It can be modelled by a part of the curve  $y = \sec^2 x$  between  $x = -\frac{\pi}{3}$  and  $x = \frac{\pi}{3}$ 

[8]

[7]

- (i) Find the exact area of the shaded region.
- PQ is the tangent to the curve at  $x = -\frac{\pi}{6}$
- (ii) Find the equation of this tangent.

## THIS IS THE END OF THE QUESTION PAPER