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**ADVANCED**
**General Certificate of Education**
**2009**

# Mathematics

Assessment Unit M3

*assessing*

Module M3: Mechanics 3

**[AMM31]**

AMM31

**MONDAY 15 JUNE, AFTERNOON**

## TIME

1 hour 30 minutes.

## INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.
Answer **all six** questions.
Show clearly the full development of your answers.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or scientific calculator in this paper.

## INFORMATION FOR CANDIDATES

The total mark for this paper is 75
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
Answers should include diagrams where appropriate and marks may be awarded for them.
Take $g = 9.8\,\text{m}\,\text{s}^{-2}$, unless specified otherwise.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that
$\ln z \equiv \log_e z$

**Answer all six questions.**

**Show clearly the full development of your answers.**

**Answers should be given to three significant figures unless otherwise stated.**

**1** Three particles of mass 1 kg, 2 kg and 3 kg are situated at fixed points whose coordinates, where $a > 0$, are shown in **Table 1** below.

**Table 1**

| Mass (kg) | Point |
|---|---|
| 1 | $(1, a^2, 3)$ |
| 2 | $(4, -a, a)$ |
| 3 | $(3, -1, 1)$ |

The centre of mass of the system is $(X, Y, Z)$.

**(i)** Find $X$. [4]

**(ii)** Given that $Y = 0$, find $a$. [4]

**(iii)** Hence find $Z$. [3]

**2** A and B are two points on a fixed horizontal beam one metre apart.
A particle P of weight 50 N is attached to two light elastic strings. The other ends of the strings are attached to A and B.
P hangs in equilibrium with AP stretched to 0.6 m and BP stretched to 0.8 m as shown in **Fig. 1** below.

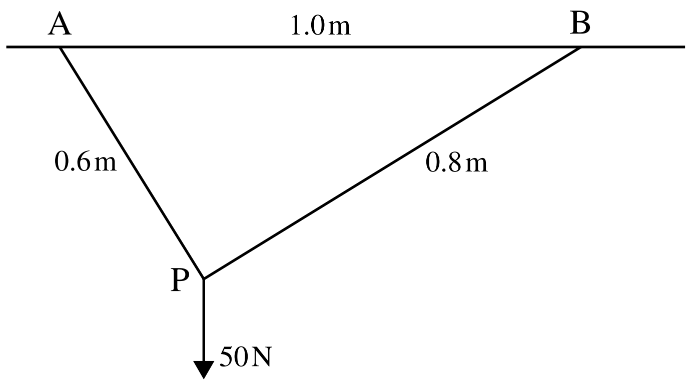


**Fig. 1**

**(i)** By resolving in the direction of AP, or otherwise, show that the tension in AP is 40 N. [4]

The natural length of the string AP is 0.5 m.

**(ii)** Find the modulus of elasticity of AP. [2]

The modulus of elasticity of the string BP is 50 N.

**(iii)** Find the natural length of BP. [4]

**3** The equation of motion of a particle moving in a straight line with S.H.M. of amplitude $a$ is given by

$$\ddot{x} = -\omega^2 x$$

where $x$ is the displacement from the centre of oscillation.

A particle P moving with S.H.M. has velocity $8\,\text{m s}^{-1}$ when $x$ is 3 m and has velocity $6\,\text{m s}^{-1}$ when $x$ is 4 m.

**(i)** Find $a$ and $\omega$ for the motion of P. [7]

A second particle Q also moving with S.H.M. has a maximum speed of $10\,\text{m s}^{-1}$ and an acceleration of maximum magnitude $50\,\text{m s}^{-2}$

**(ii)** Find $a$ and $\omega$ for the motion of Q. [4]

**4** Three forces $\mathbf{F_1}$, $\mathbf{F_2}$ and $\mathbf{F_3}$ newtons move a particle of mass 1 kg from A to B, where AB is measured in metres and

$$\overrightarrow{AB} = \begin{pmatrix} 3 \\ 6 \\ -2 \end{pmatrix}$$

**(i)** Given that

$$\mathbf{F_1} = \begin{pmatrix} 4 \\ -3 \\ 12 \end{pmatrix}$$

show that the work done by $\mathbf{F_1}$ is **not** $|\mathbf{F_1}| \times |\overrightarrow{AB}|$ [5]

The other two forces $\mathbf{F_2}$ and $\mathbf{F_3}$ are:

$$\mathbf{F_2} = \begin{pmatrix} 0 \\ 8 \\ -6 \end{pmatrix} \qquad \mathbf{F_3} = \begin{pmatrix} 2a \\ -a \\ -2a \end{pmatrix}$$

**(ii)** Find, in terms of $a$, the resultant of the three forces. [1]

The particle has speeds of $24\,\text{m s}^{-1}$ at A and $26\,\text{m s}^{-1}$ at B.

**(iii)** Use the Work–Energy Principle to find $a$. [6]

**5** **Fig. 2** below shows part of the River Nagal where the banks are parallel and 100 m apart.
The water in the river is flowing at $\frac{2}{5}\,\text{m s}^{-1}$

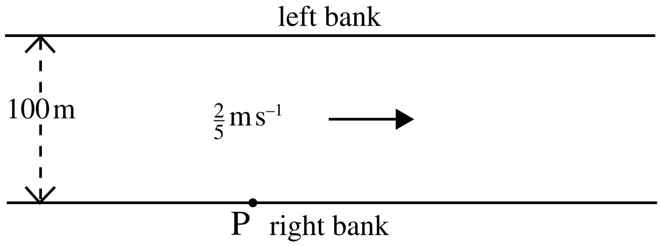


**Fig. 2**

Jean swims only on backstroke at $\frac{1}{3}\,\text{m s}^{-1}$
She is at the point P on the right bank.

Jean wishes to cross the river as quickly as possible.

**(i)** At what angle to the bank should she swim? [1]

**(ii)** Find the time taken to cross the river. [2]

**(iii)** Find how far downstream she will be when she reaches the left bank. [2]

Jean has returned to P and now wants to cross so that she lands on the left bank a shorter distance, *d* metres, downstream. She now swims at an angle $\theta°$ to the bank as shown in **Fig. 3** below.

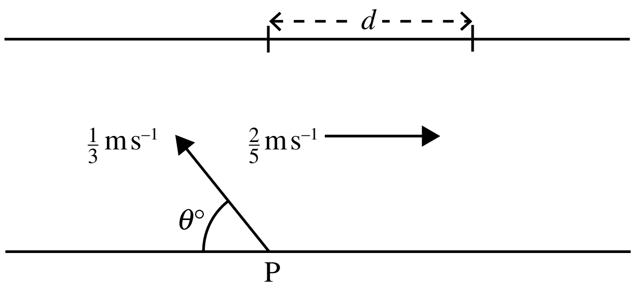


**Fig. 3**

**(iv)** Show that $d = \dfrac{20(6 - 5\cos\theta)}{\sin\theta}$ [8]

**(v)** Find *d* when $\cos\theta = \frac{5}{6}$ [2]

**(vi)** How could you check that this is a minimum value of *d*? [1]

**6** A particle P of mass 0.25 kg is moving towards the right along a straight horizontal track. The distance of P from a fixed point O on the track is $x$ metres.
Two forces act on P:
a constant frictional force of 2 N and
a variable force $F$ newtons, towards the right, where

$$F = 20 - 6\sqrt{x} \quad 0 \leqslant x \leqslant 16$$

The graph of the resultant $R$ of the two forces is shown in **Fig. 4** below.

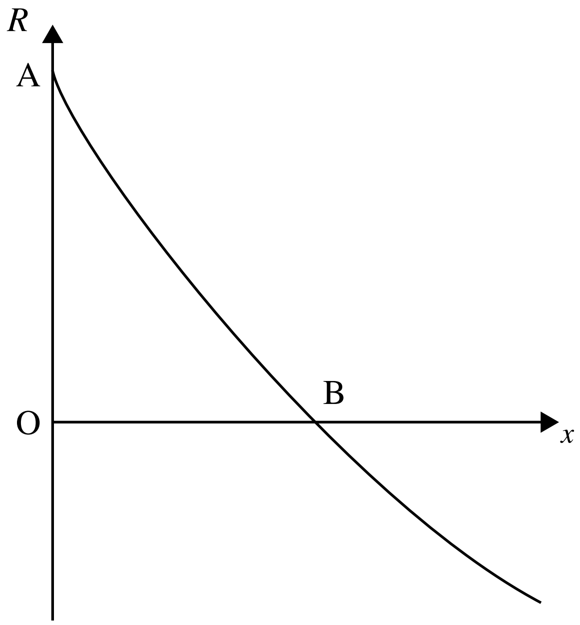


**Fig. 4**

The intercepts on the axes are A and B.

**(i)** State the coordinates of the points A and B. [3]

**(ii)** Find the work done by the resultant force $R$ for $0 \leqslant x \leqslant 16$ [5]

The speed of P at O is 12 m s$^{-1}$

**(iii)** Use the Work–Energy Principle to find the speed of the particle when $x = 16$ [3]

**(iv)** Find the maximum speed reached by the particle during this motion. [4]

**THIS IS THE END OF THE QUESTION PAPER**