



Rewarding Learning

ADVANCED
General Certificate of Education
2009

Mathematics

Assessment Unit F3

assessing

Module FP3: Further Pure Mathematics 3

[AMF31]



FRIDAY 22 MAY, MORNING

TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.

Answer **all seven** questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

You are permitted to use a graphic or a scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75

Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.

A copy of the **Mathematical Formulae and Tables booklet** is provided.

Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that

$\ln z \equiv \log_e z$

Answer all seven questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

- 1 Use the substitution $x = \frac{5}{2} \sin u$ to find

$$\int \frac{dx}{\sqrt{25-4x^2}} \quad [6]$$

- 2 Straight lines l_1 and l_2 have equations

$$l_1 \quad \frac{x-3}{2} = \frac{y-p}{3} = \frac{z-1}{-1}$$
$$l_2 \quad \frac{x-3}{1} = \frac{y+1}{-2} = \frac{z-4}{1}$$

where p is a scalar constant.

The lines intersect at the point A.

Find the value of p and the coordinates of the point A. [8]

- 3 (i) Show that

$$\frac{d}{dx} \left\{ \frac{1}{2} \left(\sin^{-1} x + x\sqrt{1-x^2} \right) \right\} = \sqrt{1-x^2} \quad [4]$$

(ii) Write $4x - x^2 - 3$ in the form $a - (x - b)^2$ [1]

(iii) Hence find the exact value of

$$\int_2^3 \sqrt{4x - x^2 - 3} \, dx \quad [5]$$

- 4 (i) Using the definition of the hyperbolic functions in terms of the exponential function, prove that

$$\cosh^2 2x + \sinh^2 2x \equiv \cosh 4x \quad [4]$$

- (ii) Hence solve the equation

$$\cosh^2 2x + \sinh^2 2x = 2$$

leaving your answers in logarithmic form. [4]

- 5 A plane Π passes through the points A (5, 3, 1), B (-3, 2, 3) and C (2, 3, 2).

- (i) Find $\vec{AC} \times \vec{BC}$. [4]

- (ii) Hence or otherwise find in Cartesian form an equation for Π . [3]

The perpendicular from the point Q(6, -6, 4) to Π meets the plane at the point P.

- (iii) Find the coordinates of P. [5]

- (iv) Show that the perpendicular distance from Q(6, -6, 4) to the plane is $2\sqrt{14}$ [2]

- 6 (a) Find the coordinates of the stationary points on the curve with equation

$$y = x - 2 \sinh^{-1} x$$

and determine their nature. [7]

- (b) Evaluate

$$\int_{-2}^0 x - 2 \sinh^{-1} x \, dx$$

correct to 2 decimal places. [7]

7 (i) Differentiate with respect to x

$$\frac{x^5}{5} (\ln x)^n \quad [3]$$

For each non-negative integer n , let

$$I_n = \int_1^e x^4 (\ln x)^n dx$$

(ii) Using your answer to (i) or otherwise, show that if $n \geq 1$, then

$$I_n = \frac{1}{5} e^5 - \frac{n}{5} I_{n-1} \quad [5]$$

The shaded region in **Fig. 1** below is bounded by the curve with equation $y = x^2 \ln x$, the line $x = e$ and the x -axis.

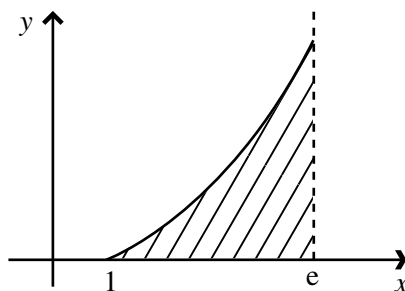


Fig. 1

The region is rotated through 2π about the x -axis.

(iii) Show that the volume of the solid formed is $\frac{\pi}{125} [17e^5 - 2]$. [7]

THIS IS THE END OF THE QUESTION PAPER
