Rewarding Learning

ADVANCED SUBSIDIARY (AS)
General Certificate of Education
2009

Mathematics

Assessment Unit F1

assessing

Module FP1: Further Pure Mathematics 1

[AMF11]

TUESDAY 23 JUNE, MORNING

AMF11

TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.
Answer **all six** questions.
Show clearly the full development of your answers.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or a scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that
$\ln z \equiv \log_e z$

Answer all six questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

1 **(a)** Describe the transformation given by the matrix

$$\mathbf{Q} = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$ [2]

(b) The matrix $\mathbf{S} = \begin{pmatrix} -1 & 1 \\ 6 & -2 \end{pmatrix}$ represents a linear transformation of the $x - y$ plane.

Find the equations of the straight lines through the origin O which are invariant under the transformation given by **S**. [6]

2 Let $\mathbf{M} = \begin{pmatrix} -1 & 1 \\ -2 & 4 \end{pmatrix}$ and $\mathbf{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

(i) Show that $\mathbf{M}^2 = 3\mathbf{M} + 2\mathbf{I}$. [4]

(ii) Hence, or otherwise, express the matrix $\mathbf{M}^4$ in the form $\alpha\mathbf{M} + \beta\mathbf{I}$ where α, β are integers. [4]

3 A binary operation $*$ is defined on the set of all ordered pairs (x, y) of real numbers, where $x \neq 0$, $y \neq 0$

The operation is given as $(a, b)*(c, d) = (ad + bc, bd)$

(i) Show that $*$ is associative. [4]

(ii) Find the identity element. [4]

(iii) Find the inverse of (a, b). [3]

4 The matrix $\mathbf{N}$ is given by $\begin{pmatrix} 2 & 0 & -6 \\ 3 & 1 & 4 \\ -1 & 0 & 1 \end{pmatrix}$

(i) Show that $\lambda = 4$ is one of the eigenvalues of $\mathbf{N}$ and find the other two eigenvalues. [7]

(ii) Find a unit eigenvector corresponding to $\lambda = 4$ [4]

5 **(a)** Find all the real values of a, b such that

$$(a + b\text{i})^2 = 21 - 20\text{i}$$ [8]

(b) **(i)** Sketch on an Argand diagram the locus of all points z such that

$$|z - \sqrt{3} - \text{i}| = \sqrt{2}$$ [3]

(ii) Hence, or otherwise, show that for all points z on the locus

$$\arg z \leqslant \frac{5\pi}{12}$$ [5]

6 The circle C_1 has equation

$$x^2 + y^2 + 2x - 14y + 40 = 0$$

(i) Find the equation of the tangent to the circle C_1 at the point (2, 6). [6]

(ii) Find the equation of the other tangent from the origin to the circle C_1 [7]

The circle C_2 has equation

$$x^2 + y^2 - 10x - 8y + 16 = 0$$

(iii) Find the points of intersection of the circles C_1 and C_2 [8]