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ADVANCED<br>General Certificate of Education<br>January 2009

## Mathematics

Assessment Unit M2
assessing
Module M2: Mechanics 2
[AMM21]


## TUESDAY 27 JANUARY, AFTERNOON

## TIME

1 hour 30 minutes.

## INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided. Answer all eight questions.
Show clearly the full development of your answers.
Answers should be given to three significant figures unless otherwise stated. You are permitted to use a graphic or a scientific calculator in this paper.

## INFORMATION FOR CANDIDATES

The total mark for this paper is 75
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
Answers should include diagrams where appropriate and marks may be awarded for them.
Take $\mathrm{g}=9.8 \mathrm{~m} \mathrm{~s}^{-2}$, unless specified otherwise.
A copy of the Mathematical Formulae and Tables booklet is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that
$\ln z \equiv \log _{\mathrm{e}} z$

## Answer all eight questions.

## Show clearly the full development of your answers.

## Answer should be given to three significant figures unless otherwise stated.

1 A truck of mass 1500 kg is travelling along a straight horizontal track at a constant speed of $5 \mathrm{~m} \mathrm{~s}^{-1}$
(i) Find the kinetic energy of the truck.

The driver applies the brakes and the truck is brought to rest.
The constant retarding force produced by the brakes is $F$ newtons.
(ii) Find the work done by $F$.

The truck travels 30 m after the brakes are applied.
(iii) Find the magnitude of $F$.

2 A particle, P , of mass 0.5 kg , moves under the action of two forces $(4 \mathbf{i}+\mathbf{j}) \mathrm{N}$ and $(-2 \mathbf{i}+\mathbf{j}) \mathrm{N}$.
(i) Find the acceleration of P .

At time $t=0$ seconds P is at the origin O and is moving with velocity $(\mathbf{i}-\mathbf{j}) \mathrm{m} \mathrm{s}^{-1}$
(ii) Find the speed of P when $t=1$
(iii) Find the position vector of P when $t=4$

3 The displacement of a particle, P , from a fixed origin O , at any time $t$ seconds, is

$$
\mathbf{r}=3 t^{2} \mathbf{i}+\left(2 t^{3}-t\right) \mathbf{j}+2 t \mathbf{k}
$$

(i) Find an expression for the velocity of P at any time $t$.
(ii) Find the initial velocity of P.
(iii) Find the acceleration of P when $t=3$
(iv) Briefly explain why the particle will not pass through O for a second time.

4 A pump in a water feature raises 100 litres of water through a height of 8 m vertically upwards every second.
The water issues as a jet with a speed of $20 \mathrm{~m} \mathrm{~s}^{-1}$
1 litre of water has a mass of 1 kg .
(i) Find the power developed by the pump.
(ii) Given that the jet of water is directed vertically upwards, find how much further the water will rise.
(iii) State one assumption you have made when answering (ii).

5 Fig. 1 below shows a box of mass $M \mathrm{~kg}$ sliding down a smooth slope from a point A to a point B.
The slope is inclined at $30^{\circ}$ to the horizontal.
The speed of the box at A is $u \mathrm{~m} \mathrm{~s}^{-1}$ and at B is $2 u \mathrm{~m} \mathrm{~s}^{-1}$
The distance AB is $d$ metres.


Fig. 1
Take the potential energy at A to be zero.
(i) Find the potential energy of the box at B.
(ii) Hence show that

$$
\begin{equation*}
u=\sqrt{\frac{\mathrm{g} d}{3}} \tag{4}
\end{equation*}
$$

6 A stone is projected horizontally with a speed of $20 \mathrm{~m} \mathrm{~s}^{-1}$ from a point 19.6 m vertically above horizontal ground as shown in Fig. 2 below.


Fig. 2
(i) Show that the time taken for the stone to hit the ground is 2 s .
(ii) Find the horizontal distance travelled by the stone before it hits the ground.
(iii) Find the direction of motion of the stone when it has been travelling for 1 s .

7 A racing car of mass 1000 kg travels around a bend in a road.
The bend has radius 50 m and is banked at $\theta^{\circ}$ to the horizontal as shown in Fig. 3 below.


Fig. 3

$$
\sin \theta^{\circ}=\frac{5}{13}
$$

The coefficient of friction between the car's tyres and the road is 0.4
The car is travelling at the maximum speed, $v \mathrm{~m} \mathrm{~s}^{-1}$, at which it can safely negotiate the bend.
Model the car as a particle.
(i) Draw a diagram showing all the external forces acting on the car.
(ii) By resolving vertically, find the normal reaction between the car and the road.
(iii) Find $v$.

## 8 Take $\mathbf{g}=10 \mathrm{~m} \mathrm{~s}^{-2}$ in this question.

A ball of mass 0.2 kg is thrown vertically upwards from ground level.
It is known that the air resistance on the ball has magnitude $0.004 v^{2}$ newtons.
At time $t$ seconds the ball is moving with velocity $v \mathrm{~m} \mathrm{~s}^{-1}$, at a displacement $x$ metres above ground level.
(i) Show that the motion of the ball can be modelled by the differential equation

$$
\begin{equation*}
v \frac{\mathrm{~d} v}{\mathrm{~d} x}=-0.02\left(500+v^{2}\right) \tag{4}
\end{equation*}
$$

At $t=0, v=15$
(ii) Find the maximum height reached by the ball.

