General Certificate of Education
June 2005
Advanced Level Examination

MATHEMATICS AND STATISTICS (SPECIFICATION B) **MBP5**
Unit Pure 5

Wednesday 22 June 2005 Afternoon Session

In addition to this paper you will require:
- an 8-page answer book;
- the AQA booklet of formulae and statistical tables.

You may use a standard scientific calculator **only**.

Time allowed: 1 hour 15 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MBP5.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.

Information
- The maximum mark for this paper is 60.
- Mark allocations are shown in brackets.

Advice
- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

1 Use the trapezium rule with four ordinates (three strips) to find an approximation to

$$\int_{1}^{2.5} (2^x - 1)\, dx$$

giving your answer to 3 significant figures. *(4 marks)*

2 (a) Obtain the first four terms of the binomial expansion of $(1 + 8x)^{\frac{1}{2}}$ in the form $1 + ax + bx^2 + cx^3$, where a, b and c are integers. *(4 marks)*

(b) State the range of values of x for which the full expansion is valid. *(1 mark)*

3 A curve has equation

$$y = -4 + \frac{1}{x^2}$$

(a) Find the equations of the asymptotes to the curve. *(2 marks)*

(b) Sketch the curve, indicating the coordinates of the points where the curve intersects the x-axis. *(3 marks)*

(c) Find an equation of the normal to the curve at the point $(1, -3)$. *(3 marks)*

4 (a) Express $\sin x + \cos x$ in the form $R\sin(x + \alpha)$, where R is a positive constant and $0 < \alpha < \frac{\pi}{2}$. *(3 marks)*

(b) Hence find the general solution, in radians, of the equation

$$\sin x + \cos x = \frac{1}{\sqrt{2}}$$ *(4 marks)*

(c) Using your answer to part (a), or otherwise, find

$$\int x(\sin x + \cos x)\, dx$$ *(4 marks)*

5 At each point (x, y) on a curve C, the gradient of the curve is given by

$$\frac{dy}{dx} = \frac{x}{y}$$

The point $P(0, -1)$ lies on C.

(a) Verify that P is a stationary point. *(1 mark)*

(b) (i) Show that $\dfrac{d^2y}{dx^2} = \dfrac{y^2 - x^2}{y^3}$. *(3 marks)*

(ii) Verify that P is a maximum point. *(1 mark)*

(c) Find the equation of the curve C, giving your answer in the form $y^2 = f(x)$. *(4 marks)*

6 The line l_1 has equation $\mathbf{r} = \begin{pmatrix} 2 \\ -1 \\ -2 \end{pmatrix} + s\begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix}$.

The line l_2 has equation $\mathbf{r} = \begin{pmatrix} 7 \\ 5 \\ 4 \end{pmatrix} + t\begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix}$.

(a) (i) Find the value of the scalar product

$$\begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix} \bullet \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix}$$ *(1 mark)*

(ii) Show that the acute angle between the lines l_1 and l_2 is 43°, correct to the nearest degree. *(3 marks)*

(b) The line l_1 intersects the plane $x + y + z = 20$ at the point Q. Find the position vector of Q. *(3 marks)*

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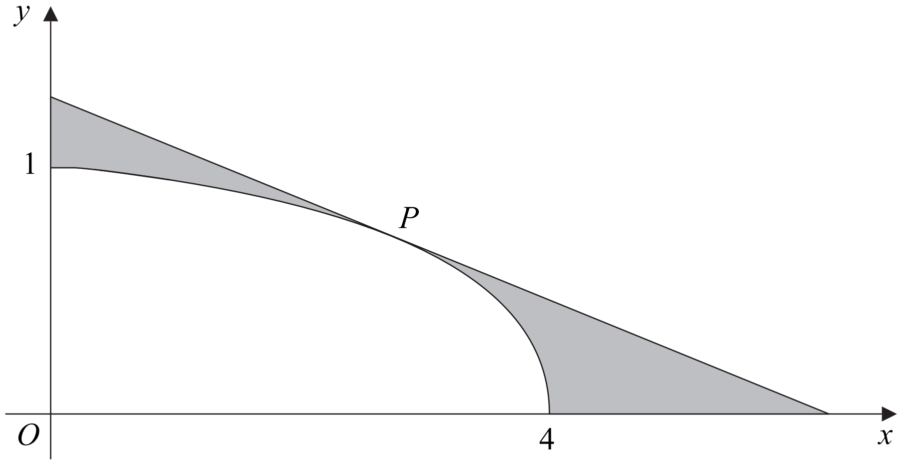

The diagram above shows the curve defined parametrically by

$$x = 4\sin t, \qquad y = \cos t, \qquad 0 \leqslant t \leqslant \frac{\pi}{2}$$

(a) Verify that $t = \frac{\pi}{2}$ gives the point (4, 0) on the curve. *(1 mark)*

(b) Show that $\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{1}{4}\tan t$. *(2 marks)*

(c) The point P on the curve is where $t = \frac{\pi}{4}$.

(i) Show that the equation of the tangent at P is $y = -\frac{1}{4}x + \sqrt{2}$. *(4 marks)*

(ii) The region bounded by the curve, the tangent and the coordinate axes is shown shaded in the diagram. Show that the area of this shaded region is given by

$$4 - 2\int_0^{\frac{\pi}{2}} 2\cos^2 t \,\mathrm{d}t$$ *(6 marks)*

(iii) Hence find the area of the shaded region, giving your answer in terms of π. *(3 marks)*

END OF QUESTIONS