General Certificate of Education
June 2005
Advanced Level Examination

MATHEMATICS AND STATISTICS **MBP4**
(SPECIFICATION B)
Unit Pure 4

Monday 20 June 2005 Morning Session

In addition to this paper you will require:
- an 8-page answer book;
- the AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 15 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MBP4.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.

Information
- The maximum mark for this paper is 60.
- Mark allocations are shown in brackets.

Advice
- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

1 Express $\dfrac{5x+3}{(x-5)(x+2)}$ in partial fractions. *(3 marks)*

2 (a) Find $\dfrac{dy}{dx}$ for each of the following:

(i) $y = (1+2x)^6$; *(2 marks)*

(ii) $y = x(1+2x)^6$. *(2 marks)*

(b) The volume, $V\,\text{m}^3$, of liquid in a container when the depth is x metres is given by

$$V = x(1+2x)^6$$

At the instant when $x = 0.5$, the depth is increasing at a rate of $0.01\,\text{m}\,\text{s}^{-1}$. Find the rate at which the volume is increasing at this instant. *(2 marks)*

(c) Find the binomial expansion of $(1+2x)^6$ in ascending powers of x up to the term in x^3, simplifying your coefficients. *(3 marks)*

3 (a) Given that $f(x) = x^5 + 5x^2 + 2$, find $f'(x)$. *(1 mark)*

(b) (i) Find $\displaystyle\int \frac{x^4+2x}{x^5+5x^2+2}\,dx$. *(2 marks)*

(ii) Hence show that $\displaystyle\int_0^1 \frac{x^4+2x}{x^5+5x^2+2}\,dx = k\ln 2$, stating the value of the constant k. *(2 marks)*

(c) The equation $x^5 + 5x^2 + 2 = 0$ has a single root α. Use the Newton-Raphson method once with first approximation $x_1 = -2$ to find a second approximation, x_2, for α, giving your answer to three significant figures. *(2 marks)*

4 (a) A sequence is defined by $u_{n+1} = \dfrac{1}{1 - u_n}, \quad u_1 = \dfrac{1}{2}.$

(i) Find u_2, u_3, u_4 and u_5. *(2 marks)*

(ii) Hence explain why the sequence is periodic and state its period. *(2 marks)*

(b) A second sequence is defined by $t_{n+1} = \dfrac{5t_n + 2}{4 + t_n}, \quad t_1 = 1.5.$

(i) Find the values of t_2 and t_3, giving your answers to three significant figures. *(2 marks)*

(ii) Given that the sequence has limit L, show that $L^2 - L - 2 = 0$.

Hence find the value of L. *(4 marks)*

5 A circle with centre C has equation $x^2 + y^2 - 4x + 18y + k = 0$, where k is a constant.

(a) (i) Find the coordinates of C. *(2 marks)*

(ii) Given that the radius of the circle is 7, find the value of k. *(2 marks)*

(b) The line l_1 has equation $3x + 4y + 5d = 0$, where d is a constant.

(i) Show that the distance from C to l_1 is $|d - 6|$. *(3 marks)*

(ii) Hence find the possible values of d so that the line l_1 is a tangent to the circle. *(2 marks)*

(iii) The line l_2 has equation $y = x - 4$. Find the acute angle between l_1 and l_2 in the form $\tan^{-1} N$, where N is a positive integer. *(3 marks)*

TURN OVER FOR THE NEXT QUESTION

6 (a) Show that the equation

$$\operatorname{cosec}^2\theta + \cot\theta = 7$$

can be written as

$$x^2 + x - 6 = 0$$

where $x = \cot\theta$. *(1 mark)*

(b) Hence, or otherwise, solve the equation

$$\operatorname{cosec}^2\theta + \cot\theta = 7$$

giving all solutions to the nearest $0.1°$ in the interval $0° < \theta < 360°$. *(6 marks)*

7 (a) (i) Differentiate $\tan 3x$ with respect to x. *(2 marks)*

(ii) Find an equation of the tangent to the curve with equation $y = 2 + \tan 3x$ at the point where $x = \dfrac{\pi}{12}$. *(3 marks)*

(b) (i) Find $\displaystyle\int (4\tan 3x + \sec^2 3x)\,\mathrm{d}x$. *(3 marks)*

(ii) Show that

$$(2 + \tan 3x)^2 \equiv 3 + 4\tan 3x + \sec^2 3x$$ *(1 mark)*

(iii) The region bounded by the curve with equation $y = 2 + \tan 3x$, the coordinate axes and the line $x = \dfrac{\pi}{9}$ is rotated completely about the x-axis to form a solid of revolution. Prove that the volume generated is $\dfrac{\pi}{3}(\sqrt{3} + \pi + 4\ln 2)$. *(3 marks)*

END OF QUESTIONS