Version: 1.0 9/9/2005

General Certificate of Education

Mathematics and Statistics 6320
Specification B

MBM6 Mechanics 6

Mark Scheme

2005 examination - June series

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Key to Mark Scheme

M	mark is for	method
m	mark is dependent on one or more M marks and is for	method
A	mark is dependent on M or m marks and is for	accuracy
B	mark is independent of M or m marks and is for	accuracy
E	mark is for	explanation
√or ft or F		follow through from previous incorrect result
cao		correct answer only
cso		correct solution only
awfw		anything which falls within
awrt		anything which rounds to
acf		any correct form
ag		answer given
sc		special case
oe		or equivalent
sf		significant figure(s)
dp		decimal place(s)
A2,1		2 or 1 (or 0) accuracy marks
–*x* ee		deduct *x* marks for each error
pi		possibly implied
sca		substantially correct approach

Abbreviations used in Marking

MC – *x*	deducted *x* marks for mis-copy
MR – *x*	deducted *x* marks for mis-read
isw	ignored subsequent working
bod	given benefit of doubt
wr	work replaced by candidate
fb	formulae book

Application of Mark Scheme

No method shown:	
Correct answer without working	mark as in scheme
Incorrect answer without working	zero marks unless specified otherwise
More than one method / choice of solution:	
2 or more complete attempts, neither/none crossed out	mark both/all fully and award the mean mark rounded down
1 complete and 1 partial attempt, neither crossed out	award credit for the complete solution only
Crossed out work	do not mark unless it has not been replaced
Alternative solution using a correct or partially correct method	award method and accuracy marks as appropriate

Mathematics and Statistics B Mechanics 6 MBM6 June 2005

Q	Solution	Marks	Total	Comments
1(a)	$\dot{r} = 2t$,	M1		Attempt at $\dot{r}$ or $\dot{\theta}$
	$\dot{\theta} = 0.1\mathrm{e}^{0.1t}$,	A1		Both correct
	When $t = 2$, $r = 12$, $\theta = \mathrm{e}^{0.2}$,			
	$\dot{r} = 4$, $\dot{\theta} = 0.1\mathrm{e}^{0.2}$,			
	Radial velocity is $\dot{r} = 4$	A1		
	Transverse velocity is $r\dot{\theta} = 1.2\,\mathrm{e}^{0.2}$	A1	4	
(b)	$\ddot{r} = 2$ $\ddot{\theta} = 0.01\mathrm{e}^{0.1t}$			
	$\ddot{r} = 2$, $\ddot{\theta} = 0.01\,\mathrm{e}^{0.2}$	M1		
	Radial acceleration is $\ddot{r} - r\dot{\theta}^2$			
	$= 2 - 12 \times (0.1\mathrm{e}^{0.2})^2$	M1		
	$= 2 - 0.12\mathrm{e}^{0.4}$	A1		
	Transverse acceleration is $2\dot{r}\dot{\theta} + r\ddot{\theta}$			
	$= 8 \times 0.1\mathrm{e}^{0.2} + 12 \times 0.01\,\mathrm{e}^{0.2}$	M1		
	$= 0.92\,\mathrm{e}^{0.2}$	A1	5	
	Total		**9**	
2	**Using forces and moments** At time t, let the sphere have rolled a distance x down the inclined plane and have an angular velocity of ω. The speed of the centre of the sphere is v where $v = r\omega$. Since the sphere does not slide			
	$v = \dot{x} = r\dot{\theta} = r\omega$	B1		
	Using '$F = ma$' along the inclined plane	M1		
	$ma = mg \sin \alpha - F$	A1		
	Using '$G = I\ddot{\theta}$' about O, the centre of the cylinder,	M1		
	$Fr = \frac{2}{3} mr^2\ddot{\theta} = \frac{2}{3} mr^2\dot{\omega}$	A1		
	$F = \frac{2}{3} mr\dot{\omega}$			
	Since $v = \dot{x} = r\dot{\theta} = r\omega$, $a = r\dot{\omega}$ $ma = mg \sin \alpha - \frac{2}{3} mr\dot{\omega}$	M1		
	$\frac{5}{3} ma = mg \sin \alpha$			
	$a = \frac{3}{5} g \sin \alpha$	A1	7	

MBM6 (cont)

Q	Solution	Marks	Total	Comments
	Alternatively **Using energy** At time t, let the cylinder have rolled a distance x down the inclined plane and have an angular velocity of ω. The speed of the centre of the cylinder is v where $v = r\omega$. Since the cylinder does not slide $v = \dot{x} = r\dot{\theta} = r\omega$	(B1)		As before
	The kinetic energy of the sphere is the kinetic energy of the linear motion of the centre of mass of the sphere plus the rotational kinetic energy of the sphere $= \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$	(M1)		
	$= \frac{1}{2}m(r\omega)^2 + \frac{1}{2}\times\frac{2}{3}mr^2\times\omega^2$ $= \frac{5}{6}mr^2\omega^2$	(A1)		
	By conservation of energy, $mg\,x\sin\alpha = \frac{5}{6}mr^2\omega^2 = \frac{5}{6}mv^2$	(M1) (A1)		
	Differentiating with respect to x, $mg\sin\alpha = \frac{\mathrm{d}}{\mathrm{d}x}(\frac{5}{6}mv^2)$ $= \frac{\mathrm{d}}{\mathrm{d}v}(\frac{5}{6}mv^2)\frac{\mathrm{d}v}{\mathrm{d}t}\frac{\mathrm{d}t}{\mathrm{d}x}$ $= \frac{5}{3}mv \times a \times \frac{1}{v}$ $= \frac{5}{3}ma$	(M1)		
	$\therefore a = \frac{3}{5}g\sin\alpha$	(A1)	(7)	
	Total		**7**	

MBM6 (cont)

Q	Solution	Marks	Total	Comments
3(a)	No slipping $\Rightarrow a\dot{\theta} = v$	B1		
	Conservation of energy			
	$\frac{1}{2}\times 5m\times(a\dot{\theta})^2+\frac{1}{2}\times 2m\times(a\dot{\theta})^2$	M1		
	$+\frac{1}{2}\times 8ma^2\times\dot{\theta}^2$	A1		
	$= g(5ma\theta - 2ma\theta)$	A1		
	$15ma^2\dot{\theta}^2 = 6mga\theta$	M1		
	$a\dot{\theta}^2 = \dfrac{2g}{5}\theta$	A1	6	
	Alternatively			
	No slipping $\Rightarrow a\dot{\theta} = v$	(B1)		
	Using '$F = ma$' for each particle			
	$5mg - T_1 = 5mf$	(M1)		
	and $T_2 - 2mg = 2mf$	(A1)		
	Using '$G = I\ddot{\theta}$'			
	$T_1a - T_2a = 8ma^2\ddot{\theta}$	(M1)		
	$3mg = 7mf + 8ma\ddot{\theta}$			
	$= 15mf$			
	$a\ddot{\theta} = \frac{g}{5}$			
	$a\dot{\theta}\ddot{\theta} = \frac{g}{5}\dot{\theta}$			
	$\frac{1}{5}a\dot{\theta}^2 = \frac{g}{5}\theta + c$,	(A1)		
	$c = 0$			
	$a\dot{\theta}^2 = \dfrac{2g}{5}\theta$	(A1)	(6)	

MBM6 (cont)

Q	Solution	Marks	Total	Comments
(b)	Differentiating with respect to t			
	$2a\dot{\theta}\ddot{\theta} = \frac{2g}{5}\dot{\theta}$	M1		
	$a\ddot{\theta} = \frac{g}{5} = f$	A1		
	Using '$F = ma$' for each particle			
	$5mg - T_1 = 5mf$	M1 A1		
	and $T_2 - 2mg = 2mf$	A1		
	Hence $\left.\begin{array}{l} 5mg - T_1 = mg \\ T_1 = 4mg \\ T_2 = \frac{12}{5}mg \end{array}\right\}$	A1		
	$\therefore T_1 : T_2 = 4mg : \frac{12}{5}mg$			
	$= 5 : 3$	A1	7	
	Alternatively			
	Using $\frac{g}{5} = f$			
	$T_1 = 5mg - 5mf$			
	$= 4\ mg$	(M1)		
		(A1)		
	$T_2 = 2mg + 2mf$			
	$= \frac{12}{5}\ mg$	(M1)		
		(A1)		
	$\therefore T_1 : T_2 = 4mg : \frac{12}{5}mg$			
	$= 5 : 3$	(A3)	(7)	
	Or			
	Using '$F = ma$' for each particle			
	$5mg - T_1 = 5mf$			
	and $T_2 - 2mg = 2mf$	(M1)		
		(A2)		
	Using '$G = I\ddot{\theta}$'			
	$T_1 a - T_2 a = 8ma^2\ddot{\theta}$	(M1)		
	$T_1 = 4\ mg$	(A1)		
	$T_2 = \frac{12}{5}\ mg$	(A1)		
	$\therefore T_1 : T_2 = 4mg : \frac{12}{5}mg$			
	$= 5 : 3$	(A1)	(7)	
	Total		**13**	

MBM6 (cont)

Q	Solution	Marks	Total	Comments
4(a)	Length of AB is $2l\cos\theta$	B1		
	Extension of spring is $2l\cos\theta - l$	B1		
	Potential energy of spring is $4mg\dfrac{(2l\cos\theta - l)^2}{2l}$ $= 2mgl(2\cos\theta - 1)^2$	M1 A1		
	Potential energy of rod, below OB, is $-mgl\cos\theta$			
	$V = 2mgl(2\cos\theta - 1)^2 - mgl\cos\theta$	A1	5	
(b)	$\dfrac{dV}{d\theta} = 2mgl \times -4\sin\theta(2\cos\theta - 1) + mgl\sin\theta$	M1 A1		
	$= mgl\sin\theta\ (1 + 8 - 16\cos\theta)$ $= 0$			
	When $\theta = 0$ or $\cos^{-1}\frac{9}{16}$ [or 0.97339]	A1 A1	4	Accept $\theta = 55.8°$
(c)	$\dfrac{dV}{d\theta} = 9\,mgl\sin\theta - 8\,mgl\sin 2\theta$			
	$\dfrac{d^2V}{d\theta^2} = 9\,mgl\cos\theta - 16\,mgl\cos 2\theta$	M1 A1		
	When $\theta = 0$, $\dfrac{d^2V}{d\theta^2} = -7mgl$ thus V has maximum thus unstable equilibrium	M1 A1		Need $-7mgl$ for A1
	When $\theta = \cos^{-1}\frac{9}{16}$, $\dfrac{d^2V}{d\theta^2} = \frac{175}{16}mgl$ [ie positive]			Accept any positive
	thus V has minimum thus stable equilibrium	A1	5	
	Total		**14**	

MBM6 (cont)

Q	Solution	Marks	Total	Comments
5(a)	M of I of element is $\frac{\delta x}{4a}mx^2$	M1		Use of ρ or $\frac{m}{4a}$
	M of I of rod = $\sum \frac{\delta x}{4a}mx^2$	M1		
	$= \int_0^{4a} \frac{mx^2}{4a}\mathrm{d}x$	M1 A1		M3 A1 if used $\int_{-2a}^{2a} \frac{mx^2}{4a}\mathrm{d}x$ and then parallel axis theorem
	$= \left[\frac{m}{4a} \times \frac{x^3}{3}\right]_0^{4a}$			
	$= \frac{16}{3}ma^2$	A1	5	
(b)(i)	M of I of rod *PQ* about end *P* is $\frac{16}{3}ma^2$	B1		
	M of I of rod *PR* about end *P* is $\frac{16}{3}ma^2$			
	M of I of quadrant *QR* about *P* is $5m(4a)^2 = 80ma^2$	B1		
	M of I of ride boat is $\frac{16}{3}ma^2 + \frac{16}{3}ma^2 + 80ma^2$	M1		
	$= \frac{272}{3}ma^2$	A1	4	
(ii)	Greatest angular velocity is when the rods are inclined at $\frac{\pi}{4}$ to the downward vertical.	M1		
	Change in potential energy for rod *PQ* is $mg(2a + a\sqrt{2})$	B1		
	Change in potential energy for rod *PR* is $mg \times a\sqrt{2}$	B1		
	Change in potential energy for quadrant *QR* is $5mg\left(\frac{8a}{\pi} + \frac{8\sqrt{2}a}{\pi}\right)$	M1 A1		Need to use $\frac{8a}{\pi}$ or $\frac{8a\sqrt{2}}{\pi}$
	Conservation of energy: $\frac{1}{2} \times \frac{272}{3}ma^2\dot{\theta}^2 =$	M1		M1 if at least one side correct
	$mga\left(2 + \sqrt{2} + \sqrt{2} + \frac{40}{\pi} + \frac{40\sqrt{2}}{\pi}\right)$	A1		
	$\dot{\theta}^2 = \frac{g}{a}\frac{3(1+\sqrt{2})(\pi+20)}{68\pi}$			
	$\dot{\theta} = \sqrt{\frac{3(1+\sqrt{2})(\pi+20)}{68\pi}\frac{g}{a}}$ or $0.886\sqrt{\frac{g}{a}}$	A1	8	
				If used 8 rather than 40 [not 5*mg* used] total mark is M3 B2 A1 for (ii)
	Total		**17**	
	TOTAL		**60**	