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General Certificate of Education

Mathematics and Statistics 6320 Specification B

MBM2 Mechanics 2

Mark Scheme

2005 examination - June series

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Key to Mark Scheme

M	mark is for	method
m	mark is dependent on one or more M marks and is for	method
A	mark is dependent on M or m marks and is for	accuracy
В	mark is independent of M or m marks and is for	accuracy
E	mark is for	explanation
√or ft or F		follow through from previous
		incorrect result
cao		correct answer only
cso		correct solution only
awfw		anything which falls within
awrt		anything which rounds to
acf		any correct form
ag		answer given
sc		special case
oe		or equivalent
sf		significant figure(s)
dp		decimal place(s)
A2,1		2 or 1 (or 0) accuracy marks
- <i>x</i> ee		deduct x marks for each error
pi		possibly implied
sca		substantially correct approach

Abbreviations used in Marking

MC-x	deducted x marks for mis-copy
MR - x	deducted x marks for mis-read
isw	ignored subsequent working
bod	given benefit of doubt
wr	work replaced by candidate
fb	formulae book

Application of Mark Scheme

No method shown:

Correct answer without working	mark as in scheme
Incorrect answer without working	zero marks unless specified otherwise
More than one method / choice of solution:	
2 or more complete attempts, neither/none crossed out	mark both/all fully and award the mean mark rounded down
1 complete and 1 partial attempt, neither crossed out	award credit for the complete solution only
Crossed out work	do not mark unless it has not been replaced
Alternative solution using a correct or partially correct method	award method and accuracy marks as appropriate

Mathematics and Statistics B Mechanics 2 MBM2 June 2005

Q	Solution	Marks	Total	Comments
1(a)	$v = \int t - \frac{t^2}{5} \mathrm{d}t$	M1		Integrating both terms
	$=\frac{t^2}{2}-\frac{t^3}{15}+c$	A1		Correct integral with or without c
	$v = 0, t = 0 \Rightarrow c = 0$	A1	3	Showing $c = 0$
	$v = \frac{t^2}{2} - \frac{t^3}{15}$			
(b)	$v(5) = \frac{5^2}{2} - \frac{5^3}{15} = 4.17 \text{ ms}^{-1}$	M1 A1	2	Substituting $t = 5$ Correct v
	$s = \int_0^5 \left(\frac{t^2}{2} - \frac{t^3}{15} \right) dt$		2	
	$3 - \int_0^{\infty} \left(\frac{2}{2} - \frac{15}{15}\right)^{\frac{1}{2}}$	M1		Integrating
	$=\left[\frac{t^3}{6} - \frac{t^4}{60}\right]_0^5$	A1		Correct expression
	$-\left[\frac{6}{6}-\frac{60}{60}\right]_0$	m1		Substitution of two limits or finding c and substituting $t = 5$
	=10.4 m	A1	4	Correct distance
	Total		9	sc for only one limit M1A1A1
2(a)(i)		M1		KE calculation using $v = 2$
	$KE = \frac{1}{2} \times 35 \times 2^2 = 70 \text{ J}$	A1	2	Correct KE
(ii)	Gain in KE = $\frac{1}{2} \times 35 \times 6^2 - 70$	M1		KE calculation using $v = 6$
	= 560 J	A1 A1	3	Correct expression for gain in KE Correct gain
(b)	$PE lost = 35 \times 9.8 \times 10 \sin 40^{\circ}$	M1		PE calculation with attempt to find height
	= 2200 J	A1	2	ag Correct PE; allow 2204
(c)	2204 - 560 = 10F	M1 A1		Energy lost = Fx including 560 Correct equation
	F = 164 N	A1	3	Correct F
				Alternative M1: Three term equation of motion
				A1:Correct equation
				A1: Correct force
(d)	1, 25, 62, 17	M1		Use of KE lost = Fx
	$\frac{1}{2} \times 35 \times 6 = Fs$	A1√		Correct equation
	$\frac{1}{2} \times 35 \times 6^2 = Fs$ $s = \frac{630}{F} = 3.83 \text{ m}$	A1√	3	Correct length
	F			Follow through <i>F</i>
				Allow 3.84 Alternative
				M1: Finding acceleration
				A1: Correct use of a constant acceleration equation
				A1: Correct length from correct working
	Total		13	

MBM2 (cont)

Q	Solution	Marks	Total	Comments
3(a)	$\mathbf{v} = 50\cos(0.1t)\mathbf{i} - 50\sin(0.1t)\mathbf{j}$	M1		Differentiating
		A1	2	Correct i component
		A1	3	Correct j and k components
(b)	(-2 (2.1.)2 (-2.1.)2	M1		Finding magnitude of <i>v</i>
	$v = \sqrt{(50\cos(0.1t)) + (-50\sin(0.1t))}$	A 1		Correct expression
	$v = \sqrt{(50\cos(0.1t))^2 + (-50\sin(0.1t))^2}$ $= \sqrt{2500(\cos^2(0.1t) + \sin^2(0.1t))}$			
	$=\sqrt{2500} = 50 \text{ ms}^{-1}$	m1	4	Use of trig identity
	- \(\sigma 2300 - 30\) His	A1	4	ag Correct speed
(c)	$\mathbf{a} = -5\sin(0.1t)\mathbf{i} - 5\cos(0.1t)\mathbf{j}$	M1		Differentiating
		A1	2	Correct acceleration
(4)		В1		Finding a
(d)	$a = \sqrt{(-5\sin(0.1t))^2 + (-5\cos(0.1t))^2} = 5$	M1		Finding a Use of $F = ma$ with their acceleration
	$F = 8000 \times 5 = 40000 \text{ N}$	A1	3	Correct force
	Total		12	
4(a)				
	R mg	B1	1	Correct diagram (to include arrows and labels)
(b)	$R\cos 60^{\circ} = mg$	M1		Resolving vertically
	R = 2mg	A1	2	ag Correct R from correct working
		711	<u> </u>	as contest it from contest working
(c)	mv^2	M1		Resolving horizontally
	$R\cos 30^\circ = \frac{r}{r}$	A1		Correct equation
	$R\cos 30^\circ = \frac{mv^2}{r}$ $r = \frac{v^2}{g\sqrt{3}}$	m1		Solving for r
	$r \equiv \frac{1}{g\sqrt{3}}$	A1	4	Correct r
			-	
	Total		7	

MBM2 (cont)

Q	Solution	Marks	Total	Comments
5(a)	$EPE = \frac{30 \times 0.8^2}{2 \times 2} = 4.8 \text{ J}$	M1		Use of EPE formula with 0.8
	$EIE = {2\times2} = 4.83$	A1	2	Correct EPE
(b)(i)	$4.8 = 0.15 \times 9.8 \times 2.8 + \frac{1}{2} \times 0.15 \times v^2$	M1		Three term energy equation
				Accept $0.684 = \frac{1}{2}0.15v^2$
		A1		Correct equation
	$v = \sqrt{\frac{4.8 - 4.116}{0.075}} = 3.02 \text{ ms}^{-1}$	m1		Solving for <i>v</i>
	$v - \sqrt{\frac{0.075}{0.075}} = 3.02 \text{ ms}$	A1	4	ag Correct v from correct working
(b)(ii)	$4.8 = 0.15 \times 9.8 \times 2.8 + 0.15 \times 9.8h$	M1		Three term energy equation using height above <i>O</i>
		A1		Accept $0.684 = mgh$ Correct equation
	$h = \frac{4.8 - 4.116}{1.47} = 0.465 \text{ m}$	A1		Correct height above <i>O</i> Accept 0.47 or 0.46
	As $0.465 < 2$ the string does not	A1	4	Correct conclusion
	become taut.	111		Alternative
				M1: Use of constant acceleration equation
				A1: Correct equation
				A1: Correct height A1: Correct conclusion
	Total		10	Tit. Concer conclusion
6(a)	$\int_{0}^{5} r(0.4r)^{2} dr$	M1		Use of appropriate expression for \overline{x}
	$\overline{x} = \frac{\int_0^{\infty} (0.137) dx}{\int_0^{\infty} (0.137) dx}$	A1		Correct expression excluding limits
	$\overline{x} = \frac{\int_0^5 x (0.4x)^2 dx}{\int_0^5 (0.4x)^2 dx}$	A1		Correct limits of integration
	$=\frac{\int_0^5 x^3 \mathrm{d}x}{\int_0^5 x^2 \mathrm{d}x}$			
	$\equiv \frac{1}{\int_0^5 x^2 dx}$			
	625/4 15	m1		Evaluation of integrals
	— <u> </u>	A1	5	ag Correct result from correct working
	$\frac{125}{3}$ 4			using both limits
(b)	Radius = 2	B1		Radius of face = 2
	$\tan \alpha = \frac{2}{3} = \frac{8}{3}$	M1		Use of tan
	$\tan \alpha = \frac{2}{5/4} = \frac{8}{5}$	M1		Use of $5 - \overline{x}$
	(8)	A1		Correct expression for $\tan \alpha$
	$\alpha = \tan^{-1}\left(\frac{8}{5}\right) = 58.0^{\circ}$	A1	5	Correct angle; allow 58°
	Total		10	

MBM2 (cont)

Q	Solution	Marks	Total	Comments
7(a)	$mg = \frac{\lambda}{l} \times \frac{l}{4}$ $\lambda = 4mg$ $m\frac{d^2x}{dt^2} = mg - T$	M1		Consideration of forces in equilibrium
	l 4	A 1	2	an Compact 1 from compact working
	$\lambda = 4mg$	A1	2	ag Correct λ from correct working
(b)(i)	$m\frac{\mathrm{d}^2 x}{\mathrm{d}x^2} = mg - T$			
		M1		Two term expression for T
	$= mg - \frac{4mg}{l} \left(x + \frac{l}{4} \right)$	A1		Correct expression for T
		M1		Three term equation of motion
	$=-\frac{4mg}{l}x$	m1		Simplifying
	$d^2x = 4g$	A 1	-	
	$\frac{\mathrm{d}^2 x}{\mathrm{d}t^2} = -\frac{4g}{l}x$	A1	5	ag Correct result from correct working
(b)(ii)	Period = $2\pi \sqrt{\frac{l}{4g}} = \pi \sqrt{\frac{l}{g}}$	M1		Identifying ω
	Period = $2\pi \sqrt{\frac{1}{4g}} = \pi \sqrt{\frac{1}{g}}$	A1	2	Correct period
	Total		9	
8(a)	$mv\frac{\mathrm{d}v}{\mathrm{d}x} = -9.8m - 0.7mv$	M1		Three term equation of motion with $v = \frac{dv}{dt}$
	dx	1711		Three term equation of motion with $\int_{0}^{\infty} dx$
	$v\frac{\mathrm{d}v}{\mathrm{d}x} = -9.8 - 0.7v$			
	dx = -0.7(v+14)	A1	2	ag Correct expression from correct
	=-0.7(v+14)	AI	2	working
<i>a</i> >				
(b)	$\int \frac{v}{v} dv = \int -0.7 dx$	M1		Separating variables and forming two
	v+14 v	1411		integrals
	$\int \frac{v}{v+14} dv = \int -0.7 dx$ $\int 1 - \frac{14}{v+14} dv = -0.7x + c$ $v - 14 \ln v+14 = -0.7x + c$ $v = 20, x = 0 \Rightarrow c = 20 - 14 \ln(34)$	3.55		
	$ v-14\ln v+14 = -0.7v+c$	M1 A1		Integrating to get $\ln (v + 14)$ term Correct integration
	$n = 20, n = 0 \Rightarrow a = 20, 141\pi(24)$	m1		Finding c
	$v - 20, x = 0 \implies c = 20 - 14 \ln(34)$	A1		Correct c
	$v - 14\ln(v + 14) = -0.7x + 20 - 14\ln(34)$			
	$0.7x = 20 - v + 14\ln(v + 14) - 14\ln(34)$			
	$x = \frac{10}{7} \left(20 - v + 14 \ln \left(\frac{v + 14}{34} \right) \right)$	A1	6	ag Correct final result from correct
				working
(c)	$x = \frac{10}{7} \left(20 + 14 \ln \left(\frac{14}{34} \right) \right) = 10.8 \text{ m}$	M1		Substituting y = 0
(c)	7 (34))	A1	2	Substituting $v = 0$ Correct value
	Total	111	10	
	TOTAL		80	