GCE 2005 January Series



# Mark Scheme

## Mathematics and Statistics B

(MBS7)

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

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### Key to Mark Scheme

<b>M</b> ma	rk is formethod
<b>m</b> ma	rk is dependent on one or more M marks and is for method
<b>A</b> ma	rk is dependent on M or m marks and is foraccuracy
<b>B</b> ma	rk is independent of M or m marks and is for method and accuracy
<b>E</b> ma	rk is for explanation
$\checkmark$ or ft or F	follow through from previous
	incorrect result
CAO	correct answer only
AWFW	anything which falls within
AWRT	anything which rounds to
AG	answer given
SC	
OE	or equivalent
A2,1	
- <i>x</i> EE	deduct <i>x</i> marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
SF	significant figure(s)
DP	decimal place(s)

## **Abbreviations used in Marking**

MC – <i>x</i>	deducted <i>x</i> marks for mis-copy
MR – <i>x</i>	
ISW	ignored subsequent working
BOD	
WR	
FB	

## **Application of Mark Scheme**

#### No method shown:

Correct answer without working	mark as in scheme
Incorrect answer without working	zero marks unless specified otherwise

More than one method/choice of solution: 2 or more complete attempts, neither/none crossed out 1 complete and 1 partial attempt, neither crossed out	mark both/all fully and award the mean mark rounded down award credit for the complete solution only			
Crossed out work	do not mark unless it has not been replaced			
Alternative solution using a correct or partially correct method	award method and accuracy marks as appropriate			

and Part         allow 0, rather than 0.5, in H <sub>0</sub> 1         H <sub>0</sub> : $\mu_A - \mu_B \neq 0.5$ B1           H <sub>1</sub> : $\mu_A - \mu_B \neq 0.5$ B1           SL $\alpha = 0.05 (5\%)$ CV $z = 1.96$ $\overline{x}_A = 3.44$ $\overline{x}_B = 2.76$ $\sigma = 0.4$ $\sigma = 0.4$ $\overline{x}_A = 3.44$ $\overline{x}_B = 2.76$ $\overline{x}_A = 3.44$ $\overline{x}_B = 2.76$ $\overline{x}_A = 3.44$ $\overline{x}_B = 2.76$ $\sigma = 0.4$ $\sigma = 0.4$ $\overline{x}_A = 3.44$ $\overline{x}_B = 2.76$ $\overline{x}_A = \frac{3.44}{\sqrt{\sqrt{\frac{\sigma^2}{n_A} + \frac{\sigma^2}{n_B}}}}$ M1         use of; allow no $\mu_0$ use of; allow no $\mu_0$ $= \frac{(3.44 - 2.76) - 0.5}{\sqrt{\sqrt{\frac{0.4^2}{20} + \frac{0.4^2}{25}}}}$ A1         allow $\mu_0 = 0$ cao $= 1.49$ to 1.51           A1 $\alpha = 1.5$ $\alpha = 1.5$	Question	Solution	Marks	Total	Comments
$ \begin{array}{ c c c c c c c c } \hline 1 & H_{0}: \ \mu_{A} - \mu_{B} = 0.5 & B1 & allow 0, rather than 0.5, in H_{0} & must be population means & must include 0.5 in H_{0} & M_{1} & CV & z = 1.96 & B1 & allow 0, rather than 0.5, in H_{0} & must be population means & must include 0.5 in H_{0} & M_{1} & Cao: (allow 1.64 to 1.65 awfw for '>' in H_{0}) & CV & z = 1.96 & M1 & allow 0, rather than 0.5, in H_{0} & must be population means & must include 0.5 in H_{0} & M_{1} & Cao: (allow 1.64 to 1.65 awfw for '>' in H_{0}) & use of; allow no \mu_{0} & allow \mu_{0} = 0 & cao & allow \mu_{0} = 0 & cao & awfw & ca = 1.49 to 1.51 & A1 & awfw & ca = 1.67 with \mu_{0} = 0 & cao & awfw & ca = 1.50 & (a = 5.67 with \mu_{0} = 0) & cao & cao & ca = 1.50 & (a = 5.67 with \mu_{0} = 0) & cao & cao & can $	Number				
$\begin{array}{ c c c c } H_{1} & \mu_{A} - \mu_{B} \neq 0.5 \\ SL & \alpha = 0.05 (5\%) \\ CV & z = 1.96 \\ \hline \\ \overline{x}_{A} = 3.44 & \overline{x}_{B} = 2.76 & \sigma = 0.4 \\ z = \frac{(\overline{x}_{A} - \overline{x}_{B}) - \mu_{0}}{\sqrt{\frac{\sigma^{2}}{n_{A}} + \frac{\sigma^{2}}{n_{B}}}} \\ = \frac{(3.44 - 2.76) - 0.5}{\sqrt{\frac{0.4^{2}}{20} + \frac{0.4^{2}}{25}}} \\ = 1.49 \text{ to } 1.51 \\ \hline \\ \end{array} \qquad \begin{array}{l} \text{must be population means} \\ \text{must include } 0.5 \text{ in } H_{0} \& H_{1} \\ \text{cao: (allow 1.64 to 1.65 awfw for '>' in H_{0})} \\ \text{use of; allow no } \mu_{0} \\ \text{use of; allow no } \mu_{0} \\ \text{allow } \mu_{0} = 0 \\ \text{cao} \\ \text{awfw} \\ \text{(ca = 1.5) (a = 5.67 \text{ with } \mu_{0} = 0)} \\ \end{array}$	and Part				
$ \begin{array}{ c c c c c c } & H_{1}: \ \mu_{A} - \mu_{B} \neq 0.5 & B1 \\ SL & \alpha = 0.05 \ (5\%) \\ CV & z = 1.96 & B1 \\ \hline \overline{x}_{A} = 3.44 & \overline{x}_{B} = 2.76 & \sigma = 0.4 \\ & z = \frac{(\overline{x}_{A} - \overline{x}_{B}) - \mu_{0}}{\sqrt{\frac{\sigma^{2}}{n_{A}} + \frac{\sigma^{2}}{n_{B}}}} & M1 \\ & = \frac{(3.44 - 2.76) - 0.5}{\sqrt{\frac{0.4^{2}}{20} + \frac{0.4^{2}}{25}}} & A1 \\ & = 1.49 \ to \ 1.51 & A1 & A1 \\ \end{array} $	1	H <sub>0</sub> : $\mu_{\rm A} - \mu_{\rm B} = 0.5$	B1		
$\begin{aligned} SL & \alpha = 0.05 (5\%) \\ CV & z = 1.96 \\ \hline R_{A} = 3.44 & \overline{x}_{B} = 2.76 & \sigma = 0.4 \\ z = \frac{(\overline{x}_{A} - \overline{x}_{B}) - \mu_{0}}{\sqrt{\frac{\sigma^{2}}{n_{A}} + \frac{\sigma^{2}}{n_{B}}}} \\ = \frac{(3.44 - 2.76) - 0.5}{\sqrt{\frac{0.4^{2}}{20} + \frac{0.4^{2}}{25}}} \\ = 1.49 \text{ to } 1.51 \end{aligned} \qquad \begin{aligned} \text{B1} \\ \text{B2} \\ \text{B1} \\ \text{B2} \\ \text{B1} \\ \text{B1} \\ \text{B2} \\ \text{B1} \\ \text{B1} \\ \text{B1} \\ \text{B2} \\ \text{B1} \\ \text{B2} \\ \text{B1} \\ \text{B1} \\ \text{B2} \\ \text{B2} \\ \text{B1} \\ \text{B2} \\ \text{B2} \\ \text{B1} \\ \text{B2} \\ \text{B1} \\ \text{B2} \\ \text{B2} \\ \text{B2} \\ \text{B2} \\ \text{B2} \\ \text{B2} \\ \text{B3} \\ \text{B2} \\ \text{B2} \\ \text{B3} \\ \text{B2} \\ \text{B3} \\ \text{B3} \\ \text{B4} \\ \text{B4} \\ \text{B4} \\ \text{B4} \\ \text{B5} \\ \text{B6} \\ \text{B1} \\ \text{B2} \\ \text{B2} \\ \text{B2} \\ \text{B2} \\ \text{B3} \\ \text{B4} \\ \text{B4} \\ \text{B5} \\ $					
CV $z = 1.96$ B1 $\bar{x}_A = 3.44$ $\bar{x}_B = 2.76$ $\sigma = 0.4$ $z = \frac{(\bar{x}_A - \bar{x}_B) - \mu_0}{\sqrt{\frac{\sigma^2}{n_A} + \frac{\sigma^2}{n_B}}}$ M1 $= \frac{(3.44 - 2.76) - 0.5}{\sqrt{\frac{0.4^2}{20} + \frac{0.4^2}{25}}}$ A1 $= 1.49$ to 1.51       A1		H <sub>1</sub> : $\mu_{\rm A} - \mu_{\rm B} \neq 0.5$	B1		must include 0.5 in $H_0 \& H_1$
$\overline{x}_{A} = 3.44  \overline{x}_{B} = 2.76  \sigma = 0.4$ $z = \frac{(\overline{x}_{A} - \overline{x}_{B}) - \mu_{0}}{\sqrt{\frac{\sigma^{2}}{n_{A}} + \frac{\sigma^{2}}{n_{B}}}}$ $M1$ $use of; allow no \mu_{0}$ $use of; allow no \mu_{0}$ $= \frac{(3.44 - 2.76) - 0.5}{\sqrt{\frac{0.4^{2}}{20} + \frac{0.4^{2}}{25}}}$ $A1$ $A1$ $use of; allow \mu_{0} = 0$ $cao$ $awfw$ $(ca = 1.5) (a = 5.67 \text{ with } \mu_{0} = 0)$					
$\overline{x}_{A} = 3.44  \overline{x}_{B} = 2.76  \sigma = 0.4$ $z = \frac{(\overline{x}_{A} - \overline{x}_{B}) - \mu_{0}}{\sqrt{\frac{\sigma^{2}}{n_{A}} + \frac{\sigma^{2}}{n_{B}}}}$ $M1$ $use of; allow no \mu_{0}$ $= \frac{(3.44 - 2.76) - 0.5}{\sqrt{\frac{0.4^{2}}{20} + \frac{0.4^{2}}{25}}}$ $A1$ $allow \mu_{0} = 0$ $cao$ $awfw$ $(ca = 1.5) (a = 5.67 \text{ with } \mu_{0} = 0)$		CV   z = 1.96	B1		cao: (allow 1.64 to 1.65 awfw for '>' in
$z = \frac{(\bar{x}_{A} - \bar{x}_{B}) - \mu_{0}}{\sqrt{\frac{\sigma^{2}}{n_{A}} + \frac{\sigma^{2}}{n_{B}}}}$ M1 use of; allow no $\mu_{0}$ $= \frac{(3.44 - 2.76) - 0.5}{\sqrt{\frac{0.4^{2}}{20} + \frac{0.4^{2}}{25}}}$ A1 allow $\mu_{0} = 0$ cao cao awfw (ca = 1.5) (a = 5.67 with $\mu_{0} = 0$ )					H <sub>0</sub> )
$= \frac{(3.44 - 2.76) - 0.5}{\sqrt{\frac{0.4^2}{20} + \frac{0.4^2}{25}}}$ $= 1.49 \text{ to } 1.51$ $A1$ $A1$ $A1$ $A1$ $A1$ $A1$ $A1$ $A$		$\overline{x}_{\rm A} = 3.44$ $\overline{x}_{\rm B} = 2.76$ $\sigma = 0.4$			
$= \frac{(3.44 - 2.76) - 0.5}{\sqrt{\frac{0.4^2}{20} + \frac{0.4^2}{25}}}$ $= 1.49 \text{ to } 1.51$ $A1$ $A1$ $A1$ $A1$ $A1$ $A1$ $A1$ $A$		$(\overline{x}_{A} - \overline{x}_{B}) - \mu_{0}$			
$= \frac{(3.44 - 2.76) - 0.5}{\sqrt{\frac{0.4^2}{20} + \frac{0.4^2}{25}}}$ $= 1.49 \text{ to } 1.51$ $A1$ $A1$ $A1$ $A1$ $A1$ $A1$ $A1$ $A$		$Z = \frac{\sqrt{X} - \frac{1}{2}}{\sqrt{\frac{\pi^2}{2} - \frac{\pi^2}{2}}}$	M1		use of; allow no $\mu_0$
$= \frac{(3.44 - 2.76) - 0.5}{\sqrt{\frac{0.4^2}{20} + \frac{0.4^2}{25}}}$ $= 1.49 \text{ to } 1.51$ $A1$ $A1$ $A1$ $A1$ $A1$ $A1$ $A1$ $A$		$\left  \frac{b}{b} + \frac{b}{b} \right $			
= 1.49 to 1.51 A1 awfw $(ca = 1.5)$ $(a = 5.67 \text{ with } \mu_0 = 0)$		$\sqrt{n_{\rm A}}$ $n_{\rm B}$			
= 1.49 to 1.51 A1 awfw $(ca = 1.5)$ $(a = 5.67 \text{ with } \mu_0 = 0)$					
= 1.49 to 1.51 A1 awfw $(ca = 1.5)$ $(a = 5.67 \text{ with } \mu_0 = 0)$		(3.44 - 2.76) - 0.5	A1		allow $\mu_0 = 0$
= 1.49 to 1.51 A1 awfw $(ca = 1.5)$ $(a = 5.67 \text{ with } \mu_0 = 0)$		$-\frac{1}{\sqrt{0.4^2 - 0.4^2}}$	A 1		
= 1.49 to 1.51 A1 awfw $(ca = 1.5)$ $(a = 5.67 \text{ with } \mu_0 = 0)$		$\sqrt{\frac{311}{20}} + \frac{311}{25}$	AI		cao
(ca = 1.5) (a = 5.67 with $\mu_0 = 0$ )			Δ1		awfw
		1.77 00 1.51	<b>A</b> 1		
		Thus, no evidence, at 5% level, to reject			$(a - 1.5) (a - 5.67 \text{ with } \mu_0 - 6)$
claim (that $\mu_A - \mu_B = 0.5$ ) A1 $\checkmark$ 8 or equivalent			A1\∕	8	or equivalent
$\begin{bmatrix} \text{claim}(\text{ulat } \mu_{\text{A}} - \mu_{\text{B}} - 0.5) \end{bmatrix} \qquad \begin{bmatrix} \text{All } v \\ \text{ft on } z \text{ and } CV \end{bmatrix}$		Channe (mat $\mu_{\rm A} - \mu_{\rm B} = 0.5)$	111 V	0	
Total 8	<u> </u>	Total		8	

#### Mathematics and Statistics B Statistics 7 MBS7 January 2005

MBS7	(cont)

`	BS7 (cont)				
Question	Solution	Marks	Total	Comments	
Number					
and Part					
2(a)	H <sub>0</sub> : $\lambda = 8$ (or $p = 0.008$ )				
	H <sub>1</sub> : $\lambda < 8$ (or $p < 0.008$ )	B1		both; no mixtures of $\lambda \& p$	
	$P(X \le 3   Po(8))$	M1		use of Po(8)	
	= 0.042  to  0.043	A1		awfw; $(ca = 0.0424)$	
	(< 5%)				
	Thus evidence, at 5% level, that average				
	number (of faulty bottles per batch) has				
	decreased	A1√	4	or equivalent	
		111	•	ft on probability versus 5%	
(b)(i)	$\sum fx = 500$			r i i i i i i i i i i i i i i i i i i i	
	$\overline{x} = \frac{\sum fx}{250} = \frac{500}{250}$	B1	1	cao ratio; ( <b>ag</b> of 2)	
(ii)	$H_0: X \sim Poisson$				
(11)	$H_1$ : not $H_0$	B1		at least H <sub>0</sub>	
		DI			
	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	M1		attempted Poisson probabilities with $\lambda = 2$	
	1 57 0.2707 67.675			I I I I I I I I I I I I I I I I I I I	
	2 74 0.2707 67.675				
	3 35 0.1804 45.100	M1		attempt at $E = 250 \times p$	
	4 28 0.0902 22.550				
	5 12 0.0361 9.025)	M1		attempt at $\geq 7 \pmod{100}$ (may be implied)	
	6 3 0.0121 3.025)				
	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	M1		attempt at combining (13.175)	
	$\chi^2 = \Sigma \frac{\left(O - E\right)^2}{E}$	MI			
	$\chi = \Sigma - E$	M1		use of	
	= 7.50 to $7.75$	A1		awfw	
	SL $\alpha = 0.01 (1\%)$				
	DF $v = 4$	B1		cao	
	or $\begin{array}{cc} CV & \chi^2 = 13.277 \\ \chi^2 = 15.086  (v = 5) \end{array}$			awfw 13.2 to 13.3	
	or $CV  \chi^2 = 15.086  (v = 5)$			awfw 15.0 to 15.1	
		B1			
	Thus no evidence, at 1% level, to reject		10		
	hypothesis that distribution is Poisson	A1√	10	or equivalent $2 + 1$ GV	
		-	17	ft on $\chi^2$ and CV	
L	Total		15		

MBS7 (cont)

MBS7 (cont)				
Question	Solution	Marks	Total	Comments
Number				
and Part				
3(a)	$\sum x = 140$ $\sum x^2 = 3500$ $\sum xy = 1587$			
	$\sum y = 63$ $\sum y^2 = 722.9$ $\overline{x} = 20$ $\overline{y} = 9$			
	$S_{xx} = 700$ $S_{yy} = 155.9$ $S_{xy} = 327$			
	$\hat{\beta} = 0.467$ $\hat{\alpha} = -0.343$ $RSS = 155.9 - \frac{327^2}{700}$	B1 B1	2	awrt
(b)(i)	$327^2$			
	$RSS = 155.9 - \frac{327}{700}$	M1		use of; even if called $s^2$
	$s^2 = \frac{RSS}{5} = 0.628 \text{ to } 0.630$	M1		use of $RSS \div 5$
	$s^2 = \frac{1}{5} = 0.628 \text{ to } 0.630$	A1	3	awfw
(ii)	H <sub>0</sub> : $\beta = 0.5$			
	H <sub>1</sub> : $\beta \neq 0.5$	B1		both
	SL $\alpha = 0.05(5\%)$	D.(		
	DF $v = 7 - 2 = 5$ CV $ t  = 2.571$	B1		cao
		B1		awrt 2.57; ignore sign
	$t = \frac{\hat{\beta} - \beta_0}{\sqrt{\frac{s^2}{S}}} = \frac{0.467 - 0.5}{\sqrt{\frac{0.629}{700}}} = -1.11 \text{ to } -1.09$	M1		use of
	$\frac{s^2}{1000000000000000000000000000000000000$	Al		awfw; ignore sign
	$\sqrt{S_{xx}}$ $\sqrt{700}$			
	Thus no evidence, at 5% level, that value	<u>,</u>		
	of $\beta$ is not 0.5	A1√	6	or equivalent
				ft on <i>t</i> and CV – consistent signs
(c)(i)	$y = -0.343 + 0.467 \times 45 = 20.5$ to 20.9	B1	1	awfw; (allow 22.1 to 22.3 awfw for use
	,			with $\beta = 0.5$ )
(ii)	$x = 45 \implies$ half-way across/middle	E1	1	or equivalent (eg 90/2)
(iii)	e	B1		or equivalent
	<i>Practical</i> : Maximum depth unlikely to bein middle of river	E1	2	on aquivalant
	or Riverbed is unlikely to be V-shaped	E1	2	or equivalent or sensible alternative
	Total		15	
4	$T \sim E(2)$		15	
(a)	1	B1	1	cao; accept 'unity'
	P(S > 5) = P(T > 4)	B1		4 cao
	$1 \left(1 - \frac{-4}{2}\right) -2$			
	$= 1 - \left(1 - e^{-\frac{4}{2}}\right) = e^{-2}$	M1		use of exponential cdf or pdf
	= 0.135	A1	3	with $\lambda = 0.5$ or 2 awrt
(c)	P(S < 5   S > 3) = P(T < 4   T > 2)	M1	3	use of conditional probability
	Exponential has 'no memory' so	M1		use of; may be implied
	= P(T < 2)	A1		2 cao; (even from $5-3$ )
	$= 1 - e^{-1} = 0.632$	A1	4	awrt
(d)		B1√	~	awfw; ft on (b)
	Implies an extremely rare event	E1	2	rare event, or equivalent
	so casts doubt on model Total		10	ag
	I OLAI		10	

Question	Solution	Marks	Total	Comments
Number				
and Part				
5(a)	Mean = 20 + 10 + 75 + 10 = 115	B1		cao
	Variance = $3^2 + 3^2 + 10^2 + 2^2$	M1		adding variances
	= 122	A1		cao; ( $\sigma$ = 11.0 to 11.1 awfw)
				(M0 A0 for $\sigma = 18$ )
	P(L + 120) = P(-7, -120 - 115)			
	$P(J < 120) = P\left(Z < \frac{120 - 115}{\sqrt{122}}\right) =$	M1		standardising 120 using ft ( $\mu \& \sigma$ )
	P(Z < 0.453) = 0.673 to 0.677	A1	5	awrt
(b)(i)	$P\left(R > \frac{2}{3}J\right) = P(3R > 2J)$	M1		use of $\frac{2}{3}$ or 3 & 2
	= P(3R > 2(C + T + R + W))			and sum of 4 parts
	= P(R > 2(C + T + W))	m1	2	cancelling of $2R$
	$\Rightarrow$ answer			ag
(ii)	Mean = 75 - 2(20 + 10 + 10) = -5	B1		cao; ignore sign
	Variance = $10^2 + 2^2(3^2 + 3^2 + 2^2)$	M1		using variances and $(-2)^2$ or $+4$
	= 188	A1		cao; ( $\sigma = 13.7 \text{ awrt}$ )
	$P(X > 0) = P\left(Z > \frac{0 - (-5)}{\sqrt{188}}\right) =$	M1		standardising 0 using ft ( $\mu \& \sigma$ )
	$P(Z > 0.36466) = 1 - \Phi(0.36466)$			
	= 0.355 to 0.360	A1	5	awfw
	Total		12	
	TOTAL		60	

MBS7 (cont)