General Certificate of Education
November 2004
Advanced Subsidiary Examination

MATHEMATICS AND STATISTICS **MBP2**
(SPECIFICATION B)
Unit Pure 2

Tuesday 2 November 2004 Morning Session

In addition to this paper you will require:
- an 8-page answer book;
- the AQA booklet of formulae and statistical tables.

You may use a standard scientific calculator **only**.

Time allowed: 1 hour 15 minutes

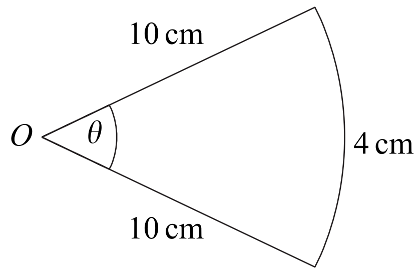
Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MBP2.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.

Information
- The maximum mark for this paper is 60.
- Mark allocations are shown in brackets.

Advice
- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

1 The diagram shows a sector of a circle, centre O and radius 10 cm. The angle of the sector is θ radians and the arc length is 4 cm.



(a) Find the value of θ. *(2 marks)*

(b) Find the area of the sector. *(2 marks)*

2 A geometric series begins

$$720 - 576 + 460.8 - \ldots$$

(a) (i) Show that the common ratio of the series is -0.8. *(1 mark)*

(ii) Give a reason why the series is convergent. *(1 mark)*

(b) Find the nth term of the series. *(2 marks)*

(c) Show that the sum of the first 15 terms of the series is approximately 414. *(2 marks)*

(d) Find the sum to infinity of the series. *(2 marks)*

3 A polynomial is given by $\mathrm{p}(x) = x^3 - 2x^2 - 11x + 12$.

(a) Use the factor theorem to show that $(x - 4)$ is a factor of $\mathrm{p}(x)$. *(2 marks)*

(b) Express $\mathrm{p}(x)$ as a product of three linear factors. *(4 marks)*

(c) Hence find all the real solutions of

$$y^6 - 2y^4 - 11y^2 + 12 = 0$$

(3 marks)

4 Find, in radians, the values of x in the interval $0 \leqslant x \leqslant 2\pi$ for which

$$\sin\left(x + \frac{\pi}{3}\right) = -0.3$$

Give your answers to 3 significant figures. *(6 marks)*

5 (a) Sketch the graph of $y = |8x|$. *(2 marks)*

(b) Sketch the graph of $y = \dfrac{1}{x^2}$, $x \neq 0$. *(2 marks)*

(c) (i) Verify that $x = \frac{1}{2}$ is a solution of the equation $\dfrac{1}{x^2} - |8x| = 0$. *(1 mark)*

(ii) The graphs of $y = \dfrac{1}{x^2}$ and $y = |8x|$ intersect at two points A and B. Find the coordinates of A and B. *(2 marks)*

6 The function f is defined for $x > 0$ by

$$\mathrm{f}(x) = \mathrm{e}^{4x} - \frac{1}{x}$$

(a) (i) Differentiate $\mathrm{f}(x)$ with respect to x to find $\mathrm{f}'(x)$. *(3 marks)*

(ii) Hence prove that f is an increasing function. *(2 marks)*

(b) Show that the area of the region bounded by the curve $y = \mathrm{e}^{4x} - \dfrac{1}{x}$, the x-axis, and the lines $x = 1$ and $x = 2$ is

$$\frac{\mathrm{e}^4(\mathrm{e}^4 - 1)}{4} - \ln 2$$

(*You may assume that this region lies entirely above the x-axis.*) *(4 marks)*

(c) The curve $y = \mathrm{e}^{4x} - \dfrac{1}{x}$ and the curve $y = 7 - \dfrac{1}{x}$ intersect at the point A. Find the x-coordinate of A, giving your answer in the form $a \ln b$, where a and b are constants to be found. *(3 marks)*

7 A curve C is defined for $x > 0$ by the equation

$$y = 2\ln x - 4x$$

The curve has a stationary point P.

(a) (i) Find $\dfrac{dy}{dx}$. *(2 marks)*

(ii) Find the gradient of the curve at the point where $x = 2$. *(1 mark)*

(b) Show that the x-coordinate of the stationary point P is $\frac{1}{2}$. *(2 marks)*

(c) (i) Find $\dfrac{d^2y}{dx^2}$. *(1 mark)*

(ii) Hence determine whether P is a maximum or a minimum point. *(2 marks)*

(d) The gradient at the point Q on the curve C is 4.

(i) Show that the x-coordinate of Q is $\frac{1}{4}$. *(2 marks)*

(ii) Find the gradient of the line PQ, giving your answer in the form $a\ln 2 + b$, where a and b are integers to be found. *(4 marks)*

END OF QUESTIONS