General Certificate of Education
June 2004
Advanced Level Examination

MATHEMATICS AND STATISTICS **MBP5**
(SPECIFICATION B)
Unit Pure 5

Wednesday 23 June 2004 Afternoon Session

In addition to this paper you will require:
- an 8-page answer book;
- the AQA booklet of formulae and statistical tables.

You may use a standard scientific calculator **only**.

Time allowed: 1 hour 15 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MBP5.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.

Information
- The maximum mark for this paper is 60.
- Mark allocations are shown in brackets.

Advice
- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

1 (a) Obtain the binomial expansion in ascending powers of x up to and including the term in x^3 of the following, giving each term in its simplest form.

(i) $(1+x)^{-1}$ *(2 marks)*

(ii) $(1+4x)^{\frac{1}{2}}$ *(4 marks)*

(b) Hence show that, for small values of x,

$$2(1+4x)^{\frac{1}{2}} + \frac{4}{1+x} \approx 6 + kx^3$$

where k is a constant to be found. *(2 marks)*

2 By considering rectangular strips of width 0.5, use the mid-ordinate rule to obtain an approximation for $\int_1^2 \frac{2}{e^{2x}-1}\,dx$, giving your answer to 3 decimal places. *(3 marks)*

3 (a) Express $6\cos x - 8\sin x$ in the form $R\cos(x+\alpha)$, where R is a positive constant and $0° < \alpha < 90°$. Give the value of α to the nearest 0.1°. *(3 marks)*

(b) Hence find the general solution, in degrees, of the equation

$$6\cos x - 8\sin x = 3$$ *(4 marks)*

4 The gradient of a curve, C, at the point (x, y) is given by

$$\frac{dy}{dx} = \frac{1}{2y(x+2)}, \qquad x > 0, \quad y > 0$$

The point $P(1, 1)$ lies on the curve C.

(a) (i) Write down the gradient of the curve C at the point P. *(1 mark)*

(ii) Show that the equation of the normal at P is $y + 6x = 7$. *(3 marks)*

(b) Find the equation of the curve C in the form $y^2 = f(x)$. *(5 marks)*

5 A curve has equation $y = \dfrac{x^2}{x+1}$.

(a) Find the equations of the two asymptotes to the curve. *(3 marks)*

(b) Given that $y \leqslant -4$ or $y \geqslant 0$ for all real values of x, and that there are no values of y for which $-4 < y < 0$, find the coordinates of the two turning points of the curve. *(3 marks)*

(c) Sketch the curve. *(3 marks)*

6 (a) Find the cosine of the angle between the vectors $\begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$ and $\begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix}$. *(3 marks)*

(b) The equations of two planes are $x - y + 2z = 0$ and $2x + 2y + z = 0$.

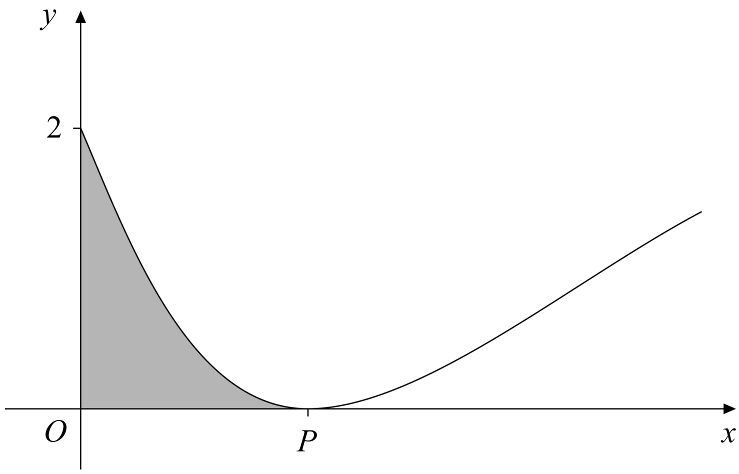
(i) Use your result from part (a) to find the acute angle between the two planes. *(2 marks)*

(ii) Verify that the point $(-5, 3, 4)$ lies in both planes. *(1 mark)*

(iii) Given that the origin also lies in both planes, write down a vector equation of the line of intersection of the two planes. *(2 marks)*

TURN OVER FOR THE NEXT QUESTION

7 The diagram shows part of a curve C.



The curve C is defined parametrically by

$$x = t^2, \qquad y = 1 + \cos t, \qquad 0 \leqslant t \leqslant 2\pi$$

The curve C touches the x-axis at the point P.

(a) Show that the x-coordinate of P is π^2. *(2 marks)*

(b) Find $\dfrac{dy}{dx}$ in terms of t. *(2 marks)*

(c) (i) Show that

$$\frac{d}{dt}\left(\frac{dy}{dx}\right) = \frac{\sin t - t\cos t}{2t^2}$$ *(2 marks)*

(ii) Hence find $\dfrac{d^2y}{dx^2}$ in terms of t. *(2 marks)*

(iii) Hence show that any point of inflection of C must have a parameter t whose value satisfies the equation $\tan t = t$. *(1 mark)*

(d) (i) Find $\int t\cos t\,dt$. *(3 marks)*

(ii) Find, in terms of π, the area of the shaded region bounded by the curve C, the y-axis and the line OP. *(4 marks)*

END OF QUESTIONS