General Certificate of Education
June 2004
Advanced Level Examination

MATHEMATICS AND STATISTICS **MBM5**
(SPECIFICATION B)
Unit Mechanics 5

Friday 25 June 2004 Morning Session

In addition to this paper you will require:
- a 12-page answer book;
- the AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 15 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MBM5.
- Answer **all** questions.
- Take $g = 9.8 \text{ m s}^{-2}$ unless stated otherwise.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.

Information
- The maximum mark for this paper is 60.
- Mark allocations are shown in brackets.

Advice
- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

1 A force, $\mathbf{F}$, acts at R, the mid-point of a light rod PQ. The coordinates of P and Q are $(3, 4, 1)$ and $(-1, 6, 9)$ respectively and $\mathbf{F} = 7\mathbf{i} - 5\mathbf{j} + 2\mathbf{k}$. The three unit vectors, $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$, are mutually perpendicular and in the directions of the x, y and z axes respectively.

Find the moment of the force $\mathbf{F}$ about the point P. *(5 marks)*

2 A ball is travelling horizontally when it hits a smooth vertical wall which is at right angles to the direction of motion of the ball. The ball is in contact with the wall for 0.2 seconds and rebounds horizontally.

The magnitude of the normal reaction force, R newtons, which acts on the ball may be modelled by

$$R = 30t(0.2 - t) \qquad 0 \leqslant t \leqslant 0.2$$

where t is the time after the ball first comes in contact with the wall.

(a) Find the magnitude of the impulse which acts on the ball. *(4 marks)*

(b) The ball has mass 5 grams and has a speed of $6\,\text{m}\,\text{s}^{-1}$ just before it hits the wall.

Find the speed with which the ball rebounds from the wall. *(4 marks)*

3 A simple pendulum consists of a particle, of mass m, fixed to one end of a light, inextensible string of length l. The other end of the string is fixed. The angle between the pendulum and the vertical is θ at time t.

(a) Prove that, for small angles of oscillation,

$$\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} = -\frac{g}{l}\theta$$ *(4 marks)*

(b) A simple pendulum has length 4 metres. The pendulum is released from rest with the string taut and at an angle of $\frac{\pi}{20}$ to the vertical.

(i) Given that $\theta = A\cos(\omega t + \alpha)$, write down the values of A, ω and α. *(3 marks)*

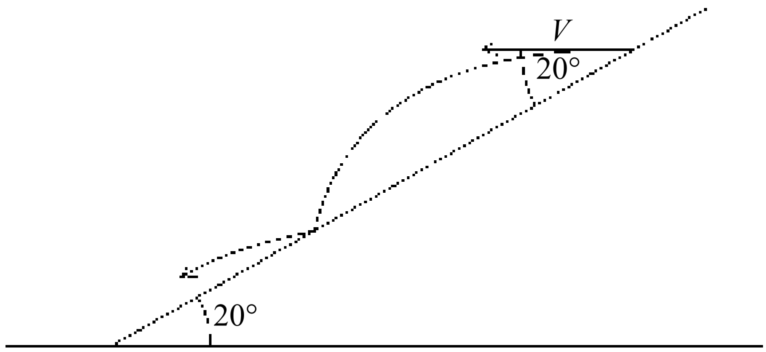
(ii) Find the time taken for the pendulum to swing through an angle of $\frac{3\pi}{40}$ from its initial position. *(5 marks)*

4 A sledge of mass M kg is set in motion sliding across a frozen horizontal lake. Snow begins to fall vertically. It falls onto the sledge, and stays on the sledge, at a constant rate of $\lambda \text{ kg s}^{-1}$. Frictional and other resistance forces acting on the sledge may be assumed to be negligible.

Show that, at time t seconds after the snow begins to fall, the speed of the sledge, $v \text{ m s}^{-1}$, satisfies the differential equation

$$(M + \lambda t)\frac{\mathrm{d}v}{\mathrm{d}t} + \lambda v = 0$$ *(6 marks)*

5 A ball is kicked down a smooth plane which is inclined at an angle of 20° to the horizontal. Initially the ball moves horizontally with speed V. The ball moves in a vertical plane containing a line of greatest slope.



(a) Find the components of the velocity of the ball, parallel and perpendicular to the plane, when it first hits the inclined plane. Give your answers in terms of V. *(9 marks)*

(b) On first striking the inclined plane, the ball bounces. The coefficient of restitution between the plane and the ball is $\frac{1}{3}$.

Show that the direction in which the ball is travelling, when it leaves the plane, makes an angle of approximately 5.5° with the plane. *(5 marks)*

TURN OVER FOR THE NEXT QUESTION

6 A particle moves along a straight line. At time t, the displacement of the particle is x. Its motion is governed by the differential equation

$$\frac{d^2x}{dt^2} + \frac{dx}{dt} = k \sin 2t$$

where k is a positive constant.

The particle starts from rest at the point $x = a$.

(a) Determine x in terms of a, k and t subject to the given initial conditions. *(11 marks)*

(b) Find the values between which x oscillates if t is so large that e^{-t} can be neglected. *(4 marks)*

END OF QUESTIONS