General Certificate of Education
June 2004
Advanced Level Examination

MATHEMATICS AND STATISTICS (SPECIFICATION B) Unit Mechanics 3

MBM3

Monday 21 June 2004 Morning Session

In addition to this paper you will require:
- a 12-page answer book;
- the AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 45 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MBM3.
- Answer **all** questions.
- Take $g = 9.8\,\mathrm{m\,s^{-2}}$ unless stated otherwise.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.

Information
- The maximum mark for this paper is 80.
- Mark allocations are shown in brackets.

Advice
- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

1 A train travels along a straight horizontal track. It starts from rest and accelerates at $0.5\,\text{m s}^{-2}$ for 10 seconds.

(a) Find the speed of the train after 10 seconds. *(2 marks)*

(b) Find the distance that the train travels in the first 10 seconds. *(2 marks)*

(c) The train has a mass of 200 tonnes and experiences a resistance force of 40 000 N. Find the magnitude of the forward driving force that acts on the train while it is accelerating at $0.5\,\text{m s}^{-2}$. *(3 marks)*

2 A particle of mass 60 kg is on a rough surface inclined at an angle of 40° to the horizontal.

(a) Find the magnitude of the normal reaction force acting on the particle. *(2 marks)*

(b) If the particle remains at rest, find the minimum value of the coefficient of friction between the particle and the slope. *(4 marks)*

(c) If the coefficient of friction between the particle and the slope is 0.2, the particle slides down the slope. Find the acceleration of the particle in this case. *(5 marks)*

3 A possible model for the acceleration, $a\,\text{m s}^{-2}$, of a particle at time t seconds is

$$a = 8 - ht$$

where h is a positive constant.

(a) The acceleration is zero when $t = 4$.

(i) Find h. *(1 mark)*

(ii) Write down an expression for a in terms of t. *(1 mark)*

(b) The velocity of the particle is $2\,\text{m s}^{-1}$ when $t = 4$. Find the velocity of the particle at time t. *(4 marks)*

4 The position vector, $\mathbf{r}$, of a particle at time t is given by

$$\mathbf{r} = 4\sin t\mathbf{i} + 4\cos t\mathbf{j} + 6t\mathbf{k}$$

The horizontal unit vectors $\mathbf{i}$ and $\mathbf{j}$ are perpendicular and the unit vector $\mathbf{k}$ is vertical.

(a) Find an expression for the velocity of the particle at time t. *(2 marks)*

(b) Find an expression for the acceleration of the particle at time t. *(2 marks)*

(c) Show that the magnitude of the acceleration of the particle is 4. *(3 marks)*

(d) Show that the speed of the particle is constant. *(3 marks)*

5 An elastic rope has natural length 4 metres and modulus of elasticity 80 N. A particle, of mass 2 kg, is attached to one end of the rope, and the other end is fixed at the point A. The particle is released from rest at A and falls vertically.

(a) When the rope just becomes taut, find:

(i) the kinetic energy of the particle; *(2 marks)*

(ii) the speed of the particle. *(3 marks)*

(b) (i) The maximum extension of the rope during the motion is x metres. Show that x satisfies the equation

$$10x^2 - 19.6x - 78.4 = 0$$ *(4 marks)*

(ii) Hence find the maximum length of the rope. *(3 marks)*

(c) State clearly **one** important assumption that you have made. *(1 mark)*

6 A boat moves so that its position vector, $\mathbf{r}$ metres, at time t seconds is given by

$$\mathbf{r} = (4t - 0.01t^2)\mathbf{i} + (5 - 3t - 0.04t^2)\mathbf{j}$$

where the unit vectors $\mathbf{i}$ and $\mathbf{j}$ are directed east and north respectively.

(a) State the position of the boat when $t = 0$. *(1 mark)*

(b) Find the time when the boat is due south of its initial position found in part (a). *(4 marks)*

(c) Find the time when the boat is travelling south east. *(5 marks)*

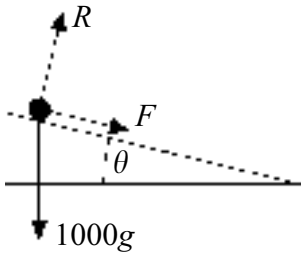
7 A car, of mass 1200 kg, is travelling up a slope at a constant speed of $20\,\mathrm{m\,s^{-1}}$. The slope is at an angle of 6° to the horizontal. A resistance force of magnitude 420 N also acts on the car when travelling at this speed. In this situation, the power output of the car is a maximum.

(a) Show that the maximum power output of the car is 33 000 W to three significant figures. *(4 marks)*

(b) The resistance force acting on the car has magnitude kv newtons, where k is a constant and $v\,\mathrm{m\,s^{-1}}$ is its speed. Find k. *(2 marks)*

(c) Find the maximum constant speed of the car on a horizontal road. *(4 mark)*

8 A car, of mass 1000 kg, travels on a banked track at a constant speed of $10\,\mathrm{m\,s^{-1}}$. The path of the car is a horizontal circle of radius 40 metres. The angle between the track and the horizontal is θ. The diagram shows the three forces acting on the car as it moves round the track, where R is the normal reaction and F is the friction. The car is modelled as a particle.



The forces all act in a vertical plane that contains the centre of the circle.

(a) The angle θ is such that $F = 0$.

(i) Show that $R = \dfrac{9800}{\cos\theta}$. *(2 marks)*

(ii) Find θ. *(5 marks)*

(b) The angle θ is reduced to 3°. The speed of the car and the radius of its circular path are unchanged. Find F. *(6 marks)*

END OF QUESTIONS