AQA

ASSESSMENT and QUALIFICATIONS ALLIANCE

Mark scheme January 2004

GCE

Mathematics & Statistics B

Unit MBP2

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Key to mark scheme

M	mark is for	method
m	mark is dependent on one or more M marks and is for	method
A	mark is dependent on M or m mark and is for	accuracy
B	mark is independent of M or m marks and is for	method and accuracy
E	mark is for	explanation
√ or ft or F		follow through from previous incorrect result
CAO		correct answer only
AWFW		anything which falls within
AWRT		anything which rounds to
AG		answer given
SC		special case
OE		or equivalent
A2,1		2 or 1 (or 0) accuracy marks
– *x* EE		Deduct *x* marks for each error
NMS		No method shown
PI		Perhaps implied
c		Candidate

Abbreviations used in marking

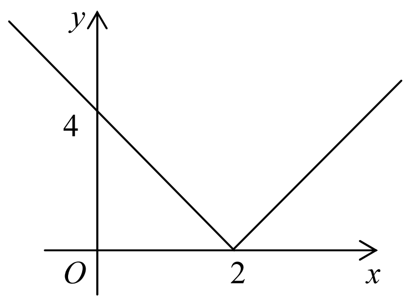
MC – *x*	deducted *x* marks for miscopy
MR – *x*	deducted *x* marks for misread
ISW	ignored subsequent working
BOD	gave benefit of doubt
WR	work replaced by candidate

Application of mark scheme

Correct answer without working	mark as in scheme
Incorrect answer without working	zero marks unless specified otherwise

Award method and accuracy marks as appropriate to an alternative solution using a correct method or partially correct method.

Question number and part	Solution	Marks	Total marks	Comments
1	$\int\left(\mathrm{e}^{2x}+4\right)\mathrm{d}x=\frac{1}{2}\mathrm{e}^{2x}+4x$	M1 A1		Either $0.5\,\mathrm{e}^{2x}$ or $k\,\mathrm{e}^{2x}+4x, k\neq 0$
	$\int_0^1\left(\mathrm{e}^{2x}+4\right)\mathrm{d}x=\frac{1}{2}\mathrm{e}^{2}+4-\frac{1}{2}$	M1		$\mathrm{F}(1)-\mathrm{F}(0)$
	$=\frac{1}{2}\left(\mathrm{e}^{2}+7\right)$	A1	4	**ag** cso
	Total		**4**	
2(a)(i)	Area of sector $=\frac{1}{2}r^2\theta$	M1		For $\frac{1}{2}r^2\theta$
	$=0.5\times 9\theta=4.5\theta\ (\mathrm{cm}^2)$	A1	2	
(ii)	Area of triangle $=\frac{1}{2}AB\times AC\sin\theta$	M1		$\frac{1}{2}AB\times AC\sin\theta$
	$\ldots=\frac{1}{2}3\times 4\sin\theta=6\sin\theta\ (\mathrm{cm}^2)$	A1	2	
(b)	{For small θ,} $\sin\theta\approx\theta$	M1		Stated or used
	Shaded area $\approx 6\theta-4.5\theta=1.5\theta\ (\mathrm{cm}^2)$	A1	2	**ag** cso
	Total		**6**	
3(a)	14 (m^3)	B1		$\frac{\mathrm{d}V}{\mathrm{d}t}=k\mathrm{e}^{-\frac{t}{12}}$
(b)	$\frac{\mathrm{d}V}{\mathrm{d}t}=-\frac{6}{12}\mathrm{e}^{-\frac{t}{12}}$	M1		isw wrong evaluation Accept equivalent statements ft (only ft if A0)
	$t=12,\ V'(t)=-0.5\,\mathrm{e}^{-1}(=-0.1839..)$	A1		
	negative sign $\Rightarrow$ Volume decreasing	E1$\sqrt{}$		
(c)	$11=8+6\mathrm{e}^{-\frac{t}{12}}$	M1		Rearrangement or $\ln 3=\ln 6+\ln\mathrm{e}^{-\frac{t}{12}}$
	$\mathrm{e}^{-\frac{t}{12}}=\frac{11-8}{6}$	m1		To the form $-\frac{t}{12}=\ln k$
	Total		**8**	

Question number and part	Solution	Marks	Total marks	Comments
4	$\tan x = -\sqrt{3}$			
	$\tan^{-1}\sqrt{3} = \frac{\pi}{3}$	M1		Inverse tangent of either $\sqrt{3}$ or $-\sqrt{3}$ attempted; can be implied by eg a correct 2nd or 4th quadrant angle
	$[x] = \pi - \frac{\pi}{3}$	m1		Angle $\left(\pi - \tan^{-1}\sqrt{3}\right)$oe in 2nd quadrant accept different forms e.g. degrees, decimals (could be negative)
	$[x] = 2\pi - \frac{\pi}{3}$	m1		Angle $\left(2\pi - \tan^{-1}\sqrt{3}\right)$ oe in 4th quadrant accept different forms e.g. degrees, decimals (could be negative if different from principal value)
	$x = \frac{2\pi}{3}$ and $x = \frac{5\pi}{3}$	A1	4	cao as the only two solutions in the interval $0 < x < 2\pi$. Ignore 'extras' outside this interval.
				sc MR solving $\tan x = \sqrt{3}$ gets max 2/4 i.e. 1st M1 and an A1 for both $\frac{\pi}{3}$ and $\frac{4\pi}{3}$
	Total		**4**	
5(a)		M1		V-shape
		A1		only in both 1st and 2nd quadrants
		A1		vertex at (2,0)
		B1	4	Graph meets y-axis at 4 only
(b)(i)	$2x - 4 = x$; or $2x - 4 = -x$; or $3x^2 - 16x + 16 = 0$;	M1		M1 for any one of the three oe
	$\Rightarrow x = 4$	A1		(If no method seen give B3 for both correct answers else B1 for one correct answer)
	$x = \frac{4}{3}$	A1	3	
(ii)	$x < \frac{4}{3}$; $x > 4$	B2,1$\surd$	2	ft on (i); penalise non-strict inequality only once
(c)	$k = 2$	M1A1	2	M1 e.g. (i) translates graph k units vertically (ii) solves $x = 2x - 4 + k$ and $x = -2x + 4 + k$ simultaneously to a single equation in k. $(12 - 3k = 4 + k)$
	Total		**11**	

Question number and part	Solution	Marks	Total marks	Comments
6(a)(i)	$2,\ 2r,\ 2r^2,\ 2r^3$	B1	1	If used a look for evidence of $a=2$ later.
(ii)	$a+ar+ar^2=ar^3=\frac{15}{4}$	M1		$3.75=\frac{a(1-r^4)}{1-r}$ gets M1
	either $4(2+2^r+2r^2+2r^3)=15$			
	or $2r^3+2r^2+2r-1.75=0$ oe	A1		'Quartic' form needs to be simplified
	$8r^3+8r^2+8r-7=0$	A1	3	**ag** cso
(b)(i)	$\mathrm{p}(0.5)=1+2+4-7$	M1		Finds value for p(0.5)
	$\ldots=0$ so $(2r-1)$ is a factor of $\mathrm{p}(r)$	A1	2	
(ii)	$(2r-1)(4r^2\ldots\ldots+7)$	M1		Valid start/end division
	$(2r-1)(4r^2+6r+7)$	A1	2	
(iii)	$\mathrm{p}(r)=0\Rightarrow 2r-1=0$ or $4r^2+6r+7=0$			
	Since $6^2<4(4)(7)$	M1		Valid consideration of Δ
	$4r^2+6r+7=0$ has no real roots {so p(r) = 0 has only 1 real solution}	A1	2	No numerical errors
(c)	$r=0.5$	B1		Can be awarded if seen in (iii)
	$S_\infty=\frac{a}{1-r};=4$	M1		
		A1	3	
	Total		**13**	

Question number and part	Solution	Marks	Total marks	Comments
7(a)(i)	$y'(x)=\frac{3}{4}x^2-\frac{6}{x}$	M1 A1	2	M1 at least 1 term correct
(ii)	$y'\left(\frac{2}{3}\right)$ $.....=\frac{1}{3}-9=-8\frac{2}{3}$	M1 A1	2	attempts to find $y'\left(\frac{2}{3}\right)$ **ag** cso
(b)(i)	$\frac{3}{4}x^2-\frac{6}{x}=0$ $3x^3=24$ $x=2$	M1 m1 A1	3	puts their $y'(x)=0$ to a single power of x Must only be the one value
(ii)	$y''(x)=\frac{3}{2}x+\frac{6}{x^2}$	M1 A1√	2	Clear differentiation of $y'(x)$ ft on equivalent forms of $y'(x)$
(iii)	$y''(2)=3+\frac{6}{4}=4.5$ $\{y''(...)>0$ so st.pt is a $\}$ minimum	A1 B1	2	**ag** cso
(c)	Gradient of $PQ=\frac{y_Q-y_P}{8-4}$ $.....=\frac{129-6\ln 8-(17-6\ln 4)}{8-4}$ $.....=\frac{129-17-6\ln\frac{8}{4}}{4}$ $.....=28-\frac{3}{2}\ln 2$	M1 m1 A1	3	y_Q-y_P numerical $\ln 8-\ln 4=\ln\frac{8}{4}$ or **both** $\ln 4=2\ln 2$ and $\ln 8=3\ln 2$ Accept $a=28$, $b=-1.5$
	Total		**14**	
	TOTAL		**60**	