

### **General Certificate of Education**

## Mathematics 6300 Specification A

MAS1/W Statistics 1

# **Mark Scheme**

### 2005 examination - June series

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Q	Solution	Marks	Total	Comments
1(a)	Length $X \sim N(48, 8^2)$			
	P(X=60)=0	B1	1	CAO
(b)	$P(X < 60) = P\left(Z < \frac{60 - 48}{8}\right)$	M1		standardising (59.5, 60 or 60.5) with 48 and $(\sqrt{8}, 8 \text{ or } 8^2)$ and/or $(48 - x)$
	= P(Z < 1.5) = 0.933	A1	2	AWRT (0.93319)
(c)	5% of 48 = 2.4	B1		CAO; OE
	P(48 - x < X < 48 + x)	M1		attempt at the probability of an interval, symmetric about 48
	= P(-0.3 < Z < 0.3)	A1		CAO 0.3
	$=\Phi(0.3)-(1-\Phi(0.3))$	m1		area change
	$= 2 \times 0.61791 - 1$			
	= 0.235 to 0.236	A1	5	AWFW (0.23582)
	Total		8	

#### MAS1/W

Q	Į	Solution	Marks	Total	Comments
	2(a)	Use of area for probability	M1		may be implied in (a) or (b)
	(i)	P(T < 10) = 0.9	A1		САО
	(ii)	P(T > 5) = 0.6	A1	3	CAO
	(b)	$P(T < 10   T > 5) = \frac{P(5 < T < 10)}{P(T > 5)}$	M1 A1		attempt at conditional probability correct form
		$=\frac{(i)-(1-(ii))}{(ii)}$	m1		
		$=\frac{0.9 - (1 - 0.6)}{0.6} = \frac{0.5}{0.6}$			
		$=\frac{5}{6}$ or 0.833	A1	4	CAO/AWRT
		OR			
		(Conditional) area above 5 is 6 <i>a</i>	(M1)		attempt at area above 5
		(Conditional) area above 5 must = $1$	(m1)		area must = 1 used
		Thus $a = \frac{1}{6}$	(A1)		
		Thus area below 10 is $(10-5)a$			
		$=\frac{5}{6}$ or 0.833	(A1)		CAO/AWRT
					Note: many candidates will quote answers without any working so:3 correct7 marks in total 2 correct in (a)2 correct in (a)3 marks in (a) 
					$\frac{0.5}{0.6}$ in (b) 3 marks in (b)
		Total		7	

Q	Solution	Marks	Total	Comments
<b>3</b> (a)	$A \sim B(15, 0.68)$			binomial only
	$P(A=10) = {\binom{15}{10}} (p)^{10} (1-p)^5$	M1		any <i>p</i> providing $0$
	$=\binom{15}{10}(0.68)^{10}(0.32)^{5}$	A1		fully correct expression
	$= 3003 \times 0.021139 \times 0.003355 = 0.213$	A1	3	AWRT
(b)	$B \sim B(40, 0.45)$			binomial only
	$P(15 \le B \le 20) = P(B \le 20 \text{ or } 19)$	M1		
	$-P(B \le 14 \text{ or } 15)$	A1		must include minus
	= 0.7870 - 0.1326 = 0.654 to 0.655	A1	3	AWFW (0.6844 / 0.2142)
	OR			
	at least 3 terms for $B(40, 0.45)$ answer	(M1) (A2)		
(c)	$C \sim B(2700, 0.25)$			Normal approx only
	Mean $(\mu) = 675$	B1		CAO
	Variance $(\sigma^2) = 506(.25)$	B1		AWRT; $SD = 22.5$ CAO
	$P(C_{\rm B} < 700) = P(C_{\rm N} < 699.5)$	B1		САО
	$= P\left(Z < \frac{699.5 - 675}{22.5}\right)$	M1		standardising (699.5, 700 or 700.5) using $\sqrt{(\mu \& \sigma)}$ ; <b>not</b> $\sigma^2$ allow (675 - x)
	= P(Z < 1.09) = 0.861 to 0.863	A1	5	AWFW
(d)	For <b>each</b> of the 15 times <b>Walk A</b> is offered select	B1		CAO
	6 numbers covering at least range 1 to	B1		1 to 60; may be implied by e.g. 2-digit
	60 Ignore repeats and numbers outside	B1	3	or (calc value) × 60 either point
	range			Note:
				Use of (a) 0.45, (b) 0.25, (c) 0.30 $\Rightarrow$
				max of M1 A1 A0, M1 A1 A0, B1 B1 B1 M1 A0, 3
	Total		14	

Q	Solution	Marks	Total	Comments
4(a)(i)	Weight, $X \sim N(\mu, 1.6^2)$			
	$95\% \Longrightarrow z = 1.96$	B1		CAO
	CI for $\mu$ is $\overline{x}(\mu) \pm z \times \frac{\sigma(s)}{\sqrt{n}}$	M1		use of; must have $(\div \sqrt{n})$ with $n > 1$
	Thus $49.2 \pm 1.96 \times \frac{1.6}{\sqrt{10}}$	A1√		$\checkmark$ on z only
	(48.2, 50.2)	A1	4	AWRT; dependent on fully correct expression
(ii)	CI includes 50 or UCL $>$ 50	B1√		$\checkmark$ on CI
	so suspicion is not supported	<b>B</b> 1√	2	$\checkmark$ on CI; dependent on first B1
(b)	Require $z \frac{\sigma}{\sqrt{n}} = \text{or} \le \text{or} < \frac{0.75}{2} \text{ or } 0.75$	M1		use of $z \frac{\sigma}{\sqrt{n}}$ with <i>n</i> unknown
	Thus $1.96 \times \frac{1.6}{\sqrt{n}} = \frac{0.75}{2} = 0.375$ or 0.75	A1√		on z; ignore signs
	Thus $n = \left(\frac{1.96 \times 1.6}{0.375}\right)^2$	m1		attempt at solving for <i>n</i>
	Thus $n = 70$	A1	4	CAO (18 with no $\div$ 2)
				Note:
				Method of 'trial and improvement': Answer = 18 CAO B2 Answer = 70 CAO B4
	Total		10	

Q	Solution	Marks	Total	Comments
5(a)	$L \sim (580, 1600)$ $G = \frac{L}{2} + 80$			
	$E(G) = \frac{580}{2} + 80 = 370$	B1		CAO
	$\mathbf{V}(G) = \frac{\mathbf{V}(L)}{4}$	M1		use of $V(aX+b) = a^2 \times V(X)$ with $a \neq 1$ and $b > 0$
	= 400	A1	3	САО
(b)(i)	R = 1000 - L - G	M1		use of
	$=920-\frac{3L}{2}$	A1	2	CAO; OE
(ii)	$E(R) = 920 - \frac{3 \times 580}{2} = 50$	B1		САО
	$V(R) = \frac{9 \times V(L)}{4}$	M1		Use of $V(aX+b) = a^2 \times V(X)$ with $a \neq \pm 1$ and $b > 0$ applied to expression for <i>R</i>
	= 3600	A1	3	CAO
				Note:
				$\sqrt{1600} + \sqrt{400} = 40 + 20 = 60$
				and $60^2 = 3600$
	Total		8	

Q	Solution	Marks	Total	Comments
6(a)(i)	$X \sim \mathbf{R}(a, a+k)$			
	$V(X) = \frac{(a+k-a)^2}{12} = \frac{k^2}{12}$	B1		CAO use of $(b-a)$ must include = $k$
	$= 48 \Rightarrow k^{2} = 576 \Rightarrow k = 24$ or $k = 24 \Rightarrow \frac{k^{2}}{12} = \frac{24^{2}}{12} = 48$	B1	2	CAO; AG
(ii)	$\mathrm{E}(X) = \frac{a+k+a}{2} = a + \frac{k}{2}$	B1		CAO; OE use of $(a+b)$ must include = $(2a+k)$ OE
	$=6 \Rightarrow a+12 = 6 \Rightarrow a = -6$ or $a = -6 \Rightarrow a + \frac{k}{2} = -6 + 12 = 6$	B1	2	CAO; AG
(b)	$P(X > 0) = \frac{a+k-0}{a+k-a} = \frac{a+k}{k}$	M1		attempt at area of a rectangle of height $\frac{1}{k}$
	$=\frac{3}{4}$ or 0.75	A1	2	CAO
(c)(i)	Mean $(\overline{X}) = 6$	B1		CAO
	$\operatorname{SE}(\overline{X}) = \sqrt{\frac{\operatorname{V}(X)}{n}}$	M1		use of; allow no $$
	= 0.8	A1	3	CAO
(ii)	Sample size is large $(n > 30)$			
	or Central Limit Theorem	B1	1	
(iii)	$P(\overline{X} > 7) = P\left(Z > \frac{7-6}{0.8}\right)$	M1		standardising 7 with answers to (i) and/or $(6-x)$
	$= P(Z > 1.25) = 1 - \Phi(1.25)$	m1		area change
	= 0.105 to $0.106$	A1	3	AWFW (0.10565)
	Total		13	
	TOTAL		60	