General Certificate of Education
June 2005
Advanced Level Examination

MATHEMATICS (SPECIFICATION A) MAP6
Unit Pure 6

AQA

Thursday 9 June 2005   Morning Session

In addition to this paper you will require:
- an 8-page answer book;
- the AQA booklet of formulae and statistical tables.

You may use a standard scientific calculator **only**.

Time allowed: 1 hour 20 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MAP6.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.
- Tie loosely any additional sheets you have used to the back of your answer book before handing it to the invigilator.

Information
- The maximum mark for this paper is 60.
- Mark allocations are shown in brackets.

Advice
- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

MAP6

Answer **all** questions.

1 (a) The matrices **A** and **B** are given by

$$\mathbf{A} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \text{and} \quad \mathbf{B} = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}.$$

Find the matrix **AB**. *(2 marks)*

(b) The matrix **M** is given by

$$\mathbf{M} = \begin{bmatrix} -3 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 3 \end{bmatrix}.$$

(i) Give a geometrical description of the transformation represented by the matrix **M**. *(4 marks)*

(ii) Which line is invariant under **M**? *(1 mark)*

2 Three simultaneous equations are given by

$$x + y \qquad = a$$
$$y + 2z = b$$
$$x - y - 3z = c \,.$$

Find, in either order:

(a) expressions for x, y and z in terms of a, b and c ;

(b) the inverse of the matrix

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 1 & -1 & -3 \end{bmatrix}.$$ *(8 marks)*

3 A transformation T is represented by the matrix $\mathbf{M}$ given by

$$\mathbf{M} = \begin{bmatrix} 1 & -2 & 3 \\ 2 & 1 & -2 \\ -5 & 1 & -1 \end{bmatrix}.$$

    (a) Find the determinant of $\mathbf{M}$.   *(2 marks)*

    (b) The volume of a given pyramid is V. Find the volume of the image of this pyramid under T. Give your answer in terms of V.   *(1 mark)*

4 The matrix $\mathbf{M}$ is given by

$$\mathbf{M} = \begin{bmatrix} 2 & -1 & 1 \\ -2 & 3 & 1 \\ 2 & 0 & 0 \end{bmatrix}.$$

    (a) Given that $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ is an eigenvector of $\mathbf{M}$, find the corresponding eigenvalue λ_1.
            (2 marks)

    (b) Given that $\lambda_2 = 4$ is an eigenvalue of $\mathbf{M}$, find a corresponding eigenvector.   *(3 marks)*

    (c) Given that $\begin{bmatrix} 1 \\ 1 \\ a \end{bmatrix}$ is the third eigenvector of $\mathbf{M}$, find a and the corresponding eigenvalue λ_3.
            (5 marks)

    (d) (i) Write down the equations of three lines which are invariant under the transformation represented by the matrix $\mathbf{M}$.   *(2 marks)*

        (ii) Show that one of these lines is perpendicular to each of the other two.   *(2 marks)*

TURN OVER FOR THE NEXT QUESTION

Turn over ▶

5 (a) Show that

$$(\mathbf{a} + \mathbf{b} - \mathbf{c}) \times (\mathbf{a} - \mathbf{b} + \mathbf{c}) = \lambda\mathbf{a} \times (\mathbf{b} - \mathbf{c}),$$

where λ is an integer to be found. *(6 marks)*

(b) The points O, A, B and C, where O is the origin, are all distinct. The position vectors of the points A, B and C are $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ respectively. Given that

$$(\mathbf{a} + \mathbf{b} - \mathbf{c}) \times (\mathbf{a} - \mathbf{b} + \mathbf{c}) = \mathbf{0},$$

show that OA is parallel to CB. *(3 marks)*

6 The four points A, B, C and D have coordinates as follows:

$$A(3, -3, -5), \quad B(6, 2, -1), \quad C(7, 3, 3) \text{ and } D(4, -2, -1).$$

(a) (i) Show that A, B, C and D are the vertices of a parallelogram. *(2 marks)*

(ii) Find $\overrightarrow{AB} \times \overrightarrow{AD}$. *(3 marks)*

(iii) Show that the area of the parallelogram is 18. *(2 marks)*

(b) The diagonals AC and BD of the parallelogram meet at the point K.

Explain why an equation of the line l through K perpendicular to the plane $ABCD$ is

$$\mathbf{r} = \begin{bmatrix} 5 \\ 0 \\ -1 \end{bmatrix} + \lambda \begin{bmatrix} 8 \\ -4 \\ -1 \end{bmatrix}.$$ *(3 marks)*

(c) The plane Π is parallel to the plane $ABCD$ and passes through the point $L(-12, 4, 9)$. Find the coordinates of M, the point of intersection of l with Π. *(6 marks)*

(d) One face of a parallelepiped is $ABCD$ and the opposite face lies in the plane Π. Find the volume of the parallelepiped. *(3 marks)*

END OF QUESTIONS