General Certificate of Education
June 2005
Advanced Level Examination

MATHEMATICS (SPECIFICATION A) **MAP4**
Unit Pure 4

Thursday 16 June 2005 Afternoon Session

In addition to this paper you will require: • an 8-page answer book; • the AQA booklet of formulae and statistical tables. You may use a standard scientific calculator **only**.

Time allowed: 1 hour 20 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MAP4.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.
- Tie loosely any additional sheets you have used to the back of your answer book before handing it to the invigilator.

Information
- The maximum mark for this paper is 60.
- Mark allocations are shown in brackets.

Advice
- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

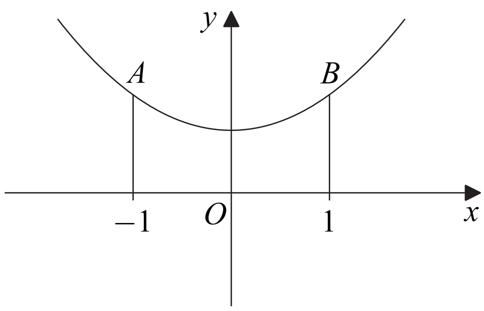
1 Solve the simultaneous equations

$$iz + 2w = 1$$

$$z - (1 + i)w = i$$

giving your answers for z and w in the form $a + ib$. *(6 marks)*

2 The diagram shows the graph of $y = \cosh x$.



(a) Show that the arc length, s, of the curve between the points A and B is given by

$$s = \int_{-1}^{1} \cosh x \, dx.$$ *(4 marks)*

(b) Hence find the value of s, giving your answer in terms of e. *(3 marks)*

3 Two loci, L_1 and L_2, in the Argand diagram, are defined by the following equations:

$$L_1 : \quad |z + 2 - 3i| = 1;$$

$$L_2 : \quad \arg(z - 4) = \tfrac{1}{2}\pi.$$

(a) Sketch the two loci on one Argand diagram. *(4 marks)*

(b) Find the smallest possible value of $|z_1 - z_2|$ where the points z_1 and z_2 lie on the loci L_1 and L_2 respectively. *(2 marks)*

4 The cubic equation

$$x^3 - 11x - 150 = 0$$

has roots α, β and γ.

(a) Write down the value of $\alpha + \beta + \gamma$. *(1 mark)*

(b) (i) Explain why

$$\alpha^3 = 11\alpha + 150 .$$ *(1 mark)*

(ii) Hence, or otherwise, show that

$$\alpha^3 + \beta^3 + \gamma^3 = 450 .$$ *(3 marks)*

(c) Given that $\alpha = -3 + 4\mathrm{i}$, write down the other non-real root β and find the third real root γ. *(2 marks)*

(d) Show that

$$(3 - 4\mathrm{i})^3 + (3 + 4\mathrm{i})^3 = -234 .$$ *(3 marks)*

5 The sequence u_1, u_2, u_3 ... is defined by

$$u_1 = 0, \qquad u_{n+1} = \tfrac{1}{2}(u_n + n) .$$

Prove by induction that, for all $n \geqslant 1$,

$$u_n = \left(\frac{1}{2}\right)^{n-1} + n - 2 .$$ *(6 marks)*

TURN OVER FOR THE NEXT QUESTION

6 (a) Sketch the curve

$$y = \tanh x,$$

indicating the asymptotes. *(2 marks)*

(b) Use the relations

$$\tanh x = \frac{\sinh x}{\cosh x} \quad \text{and} \quad \cosh^2 x - \sinh^2 x = 1$$

to show that:

(i) $\tanh^2 x = 1 - \operatorname{sech}^2 x$; *(2 marks)*

(ii) $\dfrac{d}{dx}(\tanh x) = \operatorname{sech}^2 x$. *(3 marks)*

(c) (i) Show that

$$\int_0^1 \tanh^2 x \, dx = 1 - \tanh 1 .$$ *(3 marks)*

(ii) Find

$$\int_0^1 \tanh^2 x \operatorname{sech}^2 x \, dx,$$ *(4 marks)*

giving your answer in terms of tanh 1 .

(iii) Hence find

$$\int_0^1 \tanh^4 x \, dx,$$

giving your answer in terms of tanh 1 . *(2 marks)*

7 (a) Express the complex numbers $\sqrt{3} + i$ and $2 - 2i$ in the form $re^{i\theta}$, where $r > 0$ and $-\pi < \theta \leqslant \pi$. *(4 marks)*

(b) Solve the equation

$$(2 - 2i)z^3 = \sqrt{3} + i,$$

giving each answer in the form $re^{i\theta}$, where $r > 0$ and $-\pi < \theta \leqslant \pi$. *(5 marks)*

END OF QUESTIONS