General Certificate of Education
June 2005
Advanced Level Examination

MATHEMATICS (SPECIFICATION A) **MAP3**
Unit Pure 3

Wednesday 22 June 2005 Afternoon Session

In addition to this paper you will require: • an 8-page answer book; • the AQA booklet of formulae and statistical tables. You may use a graphics calculator.

Time allowed: 1 hour 20 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MAP3.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.
- Tie loosely any additional sheets you have used to the back of your answer book before handing it to the invigilator.

Information
- The maximum mark for this paper is 60.
- Mark allocations are shown in brackets.

Advice
- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

1 (a) Write down all the terms in the binomial expansion of $(1-x)^5$. *(2 marks)*

(b) Find the coefficient of x^3 in the binomial expansion of $(3-2x)^5$. Give your answer as an integer. *(2 marks)*

2 A curve is given by the parametric equations

$$x = 2 - t^2, \qquad y = 4t.$$

(a) Find $\frac{dy}{dx}$ in terms of t. *(2 marks)*

(b) Hence find the equation of the normal to the curve at the point where $t = 4$, giving your answer in the form $y = mx + c$. *(4 marks)*

3 A biologist is studying the growth of a population of rabbits. A proposed model for the size of the population, P rabbits, t months after the study started is

$$P = 20e^{\left(\frac{t-6}{4}\right)}.$$

(a) Use this model to find, to the nearest whole number, the size of the population:

(i) after 6 months; *(1 mark)*

(ii) after 12 months. *(2 marks)*

(b) Find the time, in months, when the population first exceeds 1000 rabbits. *(3 marks)*

4 The variables x and y are related by the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{5 - x^2}{25 - x^2}.$$

(a) Starting at a point for which $x = 1$ and $y = 3$ on a solution curve, and using a step length of 0.5, find an approximate value of y when $x = 2$. Give your answer to three decimal places. *(5 marks)*

(b) Show that

$$\frac{5 - x^2}{25 - x^2} = 1 - \frac{20}{25 - x^2}.$$

(1 mark)

(c) Express

$$\frac{20}{25 - x^2} \text{ in the form } \frac{A}{5 - x} + \frac{B}{5 + x}.$$

(2 marks)

(d) (i) Find y as a function of x given that

$$y = \int \frac{5 - x^2}{25 - x^2}\,\mathrm{d}x$$

and that $y = 3$ when $x = 1$. *(5 marks)*

(ii) Find the value of y when $x = 2$. Give your answer to three decimal places. *(1 mark)*

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5 The function f is given by

$$f(x) = \frac{1}{2 - 3x}.$$

(a) (i) Find $f'(x)$ and $f''(x)$. *(4 marks)*

(ii) Hence, using the Maclaurin series, show that, for small values of x,

$$f(x) \approx \frac{1}{2} + \frac{3}{4}x + \frac{9}{8}x^2.$$ *(2 marks)*

(b) (i) Use the approximation

$$\cos x \approx 1 - \frac{x^2}{2}$$

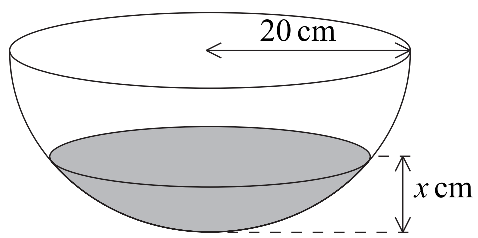
to write down a similar approximation for $\cos 2x$. *(1 mark)*

(ii) Use your results from parts (a)(ii) and (b)(i) to find an approximate solution, for small positive x, of the equation

$$\frac{1}{2 - 3x} = \cos 2x - 2x.$$

Give your answer to two decimal places. *(4 marks)*

6 A hemispherical bowl has a radius of 20 cm. The bowl is being filled with water from a tap. The depth of water in the bowl after t seconds is x cm.



The rate at which the depth of water in the bowl is increasing can be modelled by the differential equation

$$\frac{dx}{dt} = \frac{10t}{\pi(400 - x^2)}.$$

Find the time taken for the depth of water to increase from 6 cm to 20 cm. *(7 marks)*

7 A plane Π contains the points A (2, 1, −4), B (2, 4, 1) and C (6, 1, 4). The point M is the midpoint of AC.

(a) Find the vectors $\overrightarrow{AC}$ and $\overrightarrow{BM}$. *(3 marks)*

(b) Hence, or otherwise, write down an equation of the plane Π in the form $\mathbf{r} = \mathbf{a} + \lambda\mathbf{u} + \mu\mathbf{v}$. *(2 marks)*

(c) Show that the vector $\begin{bmatrix} 6 \\ 5 \\ -3 \end{bmatrix}$ is perpendicular to Π. *(4 marks)*

(d) The line BD has equation $\mathbf{r} = \begin{bmatrix} 2 \\ 4 \\ 1 \end{bmatrix} + t\begin{bmatrix} 6 \\ 5 \\ -3 \end{bmatrix}$.

Given that the lengths of BD and BM are equal, show that, at D, $t^2 = \frac{1}{5}$. *(3 marks)*

END OF QUESTIONS

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