General Certificate of Education
June 2005
Advanced Level Examination

**MATHEMATICS (SPECIFICATION A)** **MAM2/W**
**Unit Mechanics 2**

Monday 20 June 2005 Morning Session

**In addition to this paper you will require:**
- an 8-page answer book;
- the AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 20 minutes

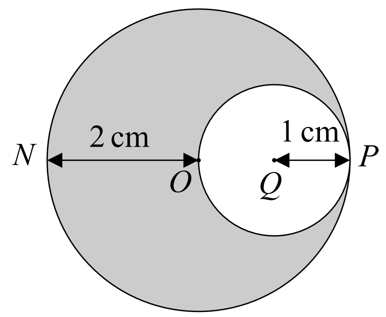
## Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MAM2/W.
- Answer **all** questions.
- Take $g = 9.8\ \mathrm{m\,s^{-2}}$ unless otherwise stated.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.
- Tie loosely any additional sheets you have used to the back of your answer book before handing it to the invigilator.

## Information
- The maximum mark for this paper is 60.
- Mark allocations are shown in brackets.

## Advice
- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

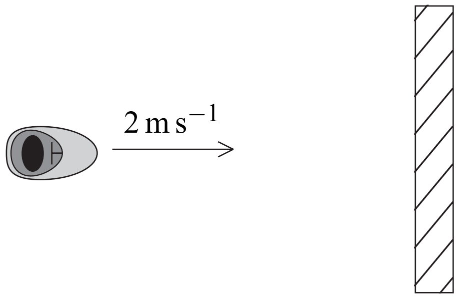
**1** The diagram shows an earring which can be modelled as part of a uniform circular lamina. It is made by removing a small circle from a larger circle, as shown in the diagram.



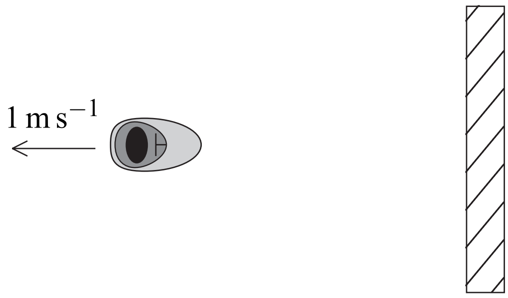

The larger circle has centre $O$ and diameter $NP$. The radius of this circle is 2 cm. The smaller circle has centre $Q$ and diameter $OP$. The radius of this circle is 1 cm.

Determine the distance of the centre of mass of the earring from $N$. *(5 marks)*

**2** At a fairground, a dodgem car is moving in a straight line, directly towards a wall, with speed $2\,\mathrm{m\,s^{-1}}$.




It hits the wall and rebounds with a speed of $1\,\mathrm{m\,s^{-1}}$.




The total mass of the car and passengers is 240 kg and they can be modelled as a single particle.

(a) Find the magnitude of the impulse given to the car as a result of the collision. *(3 marks)*

(b) A possible model for the force, $F$ newtons, between the dodgem car and the wall during contact is given by

$$F = 500t(6 - 5t), \qquad 0 \leqslant t \leqslant 1.2,$$

where $t$ denotes time in seconds.

(i) State the values of $t$ for which the force has zero magnitude. *(1 mark)*

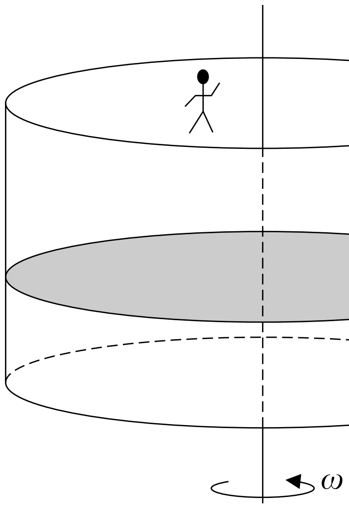
(ii) Sketch the graph of $F$ against $t$. *(2 marks)*

(iii) Show that the maximum magnitude of the force using this model is 900 N. *(3 marks)*

(iv) Briefly explain how the answer in part (a) relates to the graph in part (b)(ii). *(1 mark)*

**3** A ride in an amusement park consists of a hollow cylindrical cage of radius 3 m, with a horizontal floor and vertical sides.

Martin, who has a mass of 40 kg, stands at the edge of the floor of the cage, against the wall. The cage begins to rotate about its vertical axis. When the cage is rotating at a constant angular speed, $\omega$ radians per second, the cage floor is lowered. Martin remains in the position shown in the diagram due to the vertical frictional force between him and the wall.




(a) Draw a diagram showing the forces acting on Martin after the floor has been lowered. *(1 mark)*

(b) (i) State the magnitude of the frictional force. *(1 mark)*

(ii) The magnitude of the normal reaction between Martin and the wall is 784 N. Find the least possible value of the coefficient of friction between Martin and the wall. *(2 marks)*

(iii) Show that $\omega = 2.56$, correct to three significant figures. *(3 marks)*

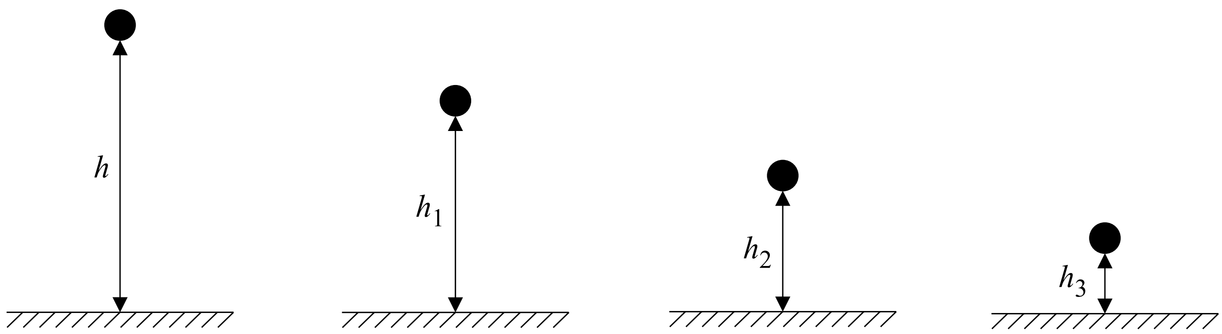
(c) State **one** modelling assumption used in this question. *(1 mark)*

**4** A small rubber ball of mass $m$ is held at height $h$ above a smooth horizontal floor and released from rest. The ball hits the floor and rebounds vertically. The coefficient of restitution between the ball and the floor is $e$.

(a) Find, in terms of $e$, $g$ and $h$, the speed with which the ball rebounds from the floor. *(3 marks)*

(b) Show that the kinetic energy of the ball immediately after it rebounds is $e^2mgh$. *(1 mark)*

(c) After the first bounce the ball rebounds to a height of $h_1$ above the floor, before returning to the floor again and rebounding to subsequent heights $h_2$ and $h_3$ above the floor, as shown in the diagrams below.




After first bounce After second bounce After third bounce

(i) Show that $h_1 = e^2h$. *(2 marks)*

(ii) Determine $h_2$ and $h_3$ in terms of $e$ and $h$. *(3 marks)*

(iii) Find, in terms of $e$ and $h$, the **total** distance travelled by the ball between being released and hitting the floor for the fourth time. *(3 marks)*

**TURN OVER FOR THE NEXT QUESTION**

**5** A single force, $\mathbf{F}$ newtons, acts on a particle of mass 0.5 kg. At time $t$ seconds, the acceleration of the particle is $\mathbf{a}$, where

$$\mathbf{a} = \begin{bmatrix} 4e^{2t} \\ 6e^{3t} \end{bmatrix} \mathrm{m\,s^{-2}}.$$

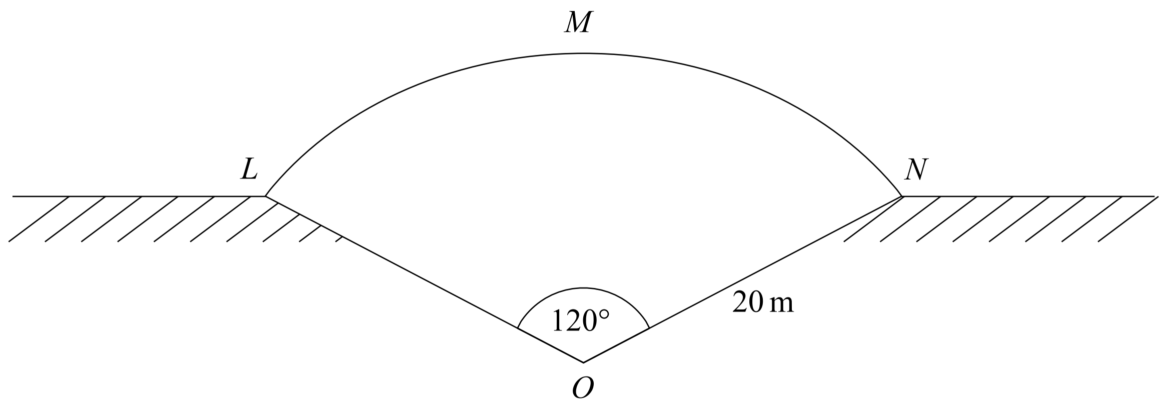
(a) Find $\mathbf{F}$ when $t = \ln 3$. *(2 marks)*

(b) Initially the velocity of the particle is $\begin{bmatrix} 1 \\ 2 \end{bmatrix} \mathrm{m\,s^{-1}}$. Show that the velocity of the particle at time $t$ seconds is

$$\begin{bmatrix} 2e^{2t} - 1 \\ 2e^{3t} \end{bmatrix}.$$ *(5 marks)*

(c) Find the power of $\mathbf{F}$ when $t = \ln 3$. *(4 marks)*

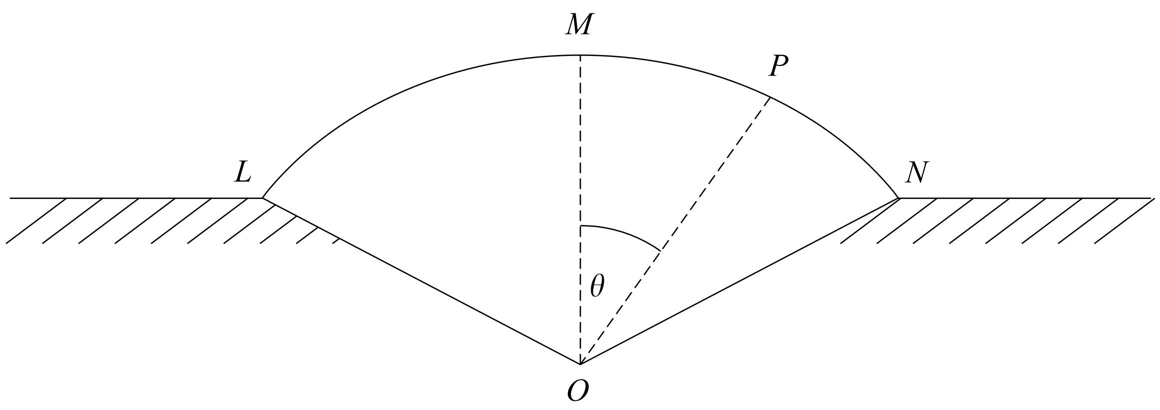
**6** A cyclist approaches a hump-backed bridge which is in the form of a circular arc *LMN*. The radius of the arc is 20 m and its centre is *O*. The angle between *OL* and *ON* is 120°. The points *L* and *N* are on the same horizontal level and *M* is the highest point of the bridge, as shown in the diagram.




The total mass of the cycle and cyclist is 60 kg. They may be modelled as a single particle. Assume that there is no resistance to the motion.

(a) Calculate the gain in potential energy as the cyclist moves from *L* to *M*. *(3 marks)*

(b) At *M* the speed of the cyclist is $6\,\mathrm{m\,s^{-1}}$. He begins to freewheel from *M* towards *N*. The point *P* is on the bridge between *M* and *N* and angle $MOP = \theta$, as shown in the diagram.




When the cyclist is at the point *P* his speed is $v$.

(i) Show that $v^2 = 428 - 392\cos\theta$. *(4 marks)*

(ii) Find, in terms of $\theta$, the normal reaction between the bridge and the cycle at *P*. *(4 marks)*

(iii) Determine whether or not the cycle remains in contact with the bridge throughout the descent from *M* to *N*. *(3 marks)*

**END OF QUESTIONS**

**THERE ARE NO QUESTIONS PRINTED ON THIS PAGE**