

General Certificate of Education
January 2005
Advanced Level Examination



MATHEMATICS (SPECIFICATION A)
Unit Statistics 3

MAS3

Monday 31 January 2005 Morning Session

In addition to this paper you will require:

- an 8-page answer book;
- the AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 20 minutes

Instructions

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MAS3.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.
- Tie loosely any additional sheets you have used to the back of your answer book before handing it to the invigilator.

Information

- The maximum mark for this paper is 60.
- Mark allocations are shown in brackets.

Advice

- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

- 1 Vikram grows tomato plants of the same variety in two different composts, A and B. The weights of tomatoes from plants grown in each compost are normally distributed with standard deviation 5 grams.

Vikram suspects that plants grown in compost A produce heavier tomatoes on average than those grown in compost B. To investigate this suspicion he takes a random sample of 10 tomatoes from plants grown in compost A and records the weight, x grams, of each tomato. He also takes a random sample of 8 tomatoes from plants grown in compost B and records the weight, y grams, of each tomato.

Vikram finds that the difference between the means of the two samples, $\bar{x} - \bar{y}$, is 3.75.

Use a test, at the 5% level of significance, to investigate Vikram's suspicion. (8 marks)

- 2 A beekeeper sells jars of honey which are labelled, "Total weight: 300 grams". She takes a random sample of 10 filled jars and records the weight, x grams, of each filled jar.

Her results are summarised below, with \bar{x} denoting the sample mean.

$$\sum x = 3030 \qquad \sum (x - \bar{x})^2 = 148$$

- (a) Calculate unbiased estimates of the mean, μ , and the variance, σ^2 , of the weight, X grams, of a jar of honey. (2 marks)
- (b) Assuming that the weights of jars of honey are normally distributed, construct a 98% confidence interval for:
- (i) the mean, μ ; (5 marks)
- (ii) the standard deviation, σ . (5 marks)
- (c) A customer claims that the jar of honey he bought weighed only 270 grams.

Use your confidence intervals, calculated in part (b), to comment on his claim.

(3 marks)

3 The random variable T has an exponential distribution with mean 5.

(a) (i) Specify fully the probability density function, $f(t)$, for T . (2 marks)

(ii) Show that, for $t \geq 0$, T has distribution function

$$F(t) = 1 - e^{-0.2t}. \quad (2 \text{ marks})$$

(iii) Find the probability that a randomly chosen value of T lies between 2 and 8. (3 marks)

(b) A roadside snack bar opens at 7.30 am each day. There is a probability of 0.15 that at least one customer is already waiting when the snack bar opens. Otherwise the time, T minutes, until the first customer arrives can be modelled by an exponential distribution with mean 5.

(i) On a particular day, there are no customers waiting when the snack bar opens.

Find the probability that no customers have arrived by 7.40 am. (2 marks)

(ii) For a randomly selected day, find the probability that the first customer arrives before 7.35 am. (3 marks)

4 A therapist works with chronically ill patients. He believes that self-hypnosis can help such patients in the relief of pain.

The therapist takes a random sample of 12 patients and teaches them techniques of self-hypnosis. Two weeks later he asks them what change there has been in their levels of pain. The results are as follows.

Patient	1	2	3	4	5	6	7	8	9	10	11	12
Less pain	✓	✓		✓	✓	✓	✓		✓	✓		✓
No change											✓	
More pain			✓					✓				

(a) (i) Name the test that you would use to investigate the therapist's belief. (1 mark)

(ii) Give a reason for your choice of test. (1 mark)

(b) Using the 5% level of significance, carry out the test that you named in part (a)(i) to investigate the therapist's belief. (7 marks)

- 5 Katy reads two newspapers, the Courier and the Journal. She always completes the crossword in each newspaper. Her completion times, X minutes for a Courier crossword and Y minutes for a Journal crossword, are normally distributed random variables.

Katy records her completion time, x minutes, for each of a random sample of 7 Courier crosswords, and her completion time, y minutes, for each of a random sample of 9 Journal crosswords. She calculates unbiased estimates of the means, μ_X and μ_Y , and the variances, σ_X^2 and σ_Y^2 , of the variables X and Y , with the following results.

	Unbiased estimate of mean	Unbiased estimate of variance
Courier crosswords	$\bar{x} = 19.4$	$s_X^2 = 9.88$
Journal crosswords	$\bar{y} = 15.9$	$s_Y^2 = 6.24$

- (a) Use a test at the 10% significance level to show that it is reasonable to believe that $\sigma_X = \sigma_Y$. *(6 marks)*
- (b) (i) Assuming that $\sigma_X = \sigma_Y$, construct a 95% confidence interval for the difference in means, $\mu_X - \mu_Y$. *(8 marks)*
- (ii) Katy believes that she is quicker, on average, at solving Journal crosswords than at solving Courier crosswords.

Use your confidence interval to comment on her belief. *(2 marks)*

END OF QUESTIONS