General Certificate of Education
January 2005
Advanced Level Examination

MATHEMATICS (SPECIFICATION A) **MAP2**
Unit Pure 2

Friday 21 January 2005 Afternoon Session

In addition to this paper you will require:
- an 8-page answer book;
- the AQA booklet of formulae and statistical tables.

You may use a standard scientific calculator **only**.

Time allowed: 1 hour 20 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MAP2.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.
- Tie loosely any additional sheets you have used to the back of your answer book before handing it to the invigilator.

Information
- The maximum mark for this paper is 60.
- Mark allocations are shown in brackets.

Advice
- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

1 The quadratic equation $x^2 - 3x + 9 = 0$ has roots α and β.

(a) Write down the numerical values of:

(i) $\alpha + \beta$; *(1 mark)*

(ii) $\alpha\beta$. *(1 mark)*

(b) Hence find the numerical values of:

(i) $\frac{6}{\alpha} \times \frac{6}{\beta}$; *(1 mark)*

(ii) $\frac{6}{\alpha} + \frac{6}{\beta}$. *(2 marks)*

(c) Hence, or otherwise, find the quadratic equation with roots $\frac{6}{\alpha}$ and $\frac{6}{\beta}$, in the form $x^2 + bx + c = 0$, where b and c are integers. *(2 marks)*

2 (a) Show that the equation $xe^x - 5 = 0$ has a root in the interval $1 < x < 2$. *(2 marks)*

(b) Differentiate xe^x with respect to x. *(2 marks)*

(c) Using the Newton–Raphson method **once**, with an initial value for x of 1.2, find an approximation to the root of the equation $xe^x - 5 = 0$ in the interval $1 < x < 2$, giving your answer to three decimal places. *(3 marks)*

3 The function f is given by

$$f(x) = x^3 + ax^2 + bx + 6.$$

When $f(x)$ is divided by $(x - 1)$, the remainder is 24.
When $f(x)$ is divided by $(x + 2)$, the remainder is also 24.

Use the Remainder Theorem to find the numerical values of a and b. *(4 marks)*

4 (a) (i) Differentiate $\ln(1 + x^2)$ with respect to x. *(2 marks)*

(ii) Hence, or otherwise, evaluate $\int_0^1 \frac{x}{1 + x^2} \, dx$. *(2 marks)*

(b) (i) Given that $y = \tan^{-1} x$, express x in terms of y. *(1 mark)*

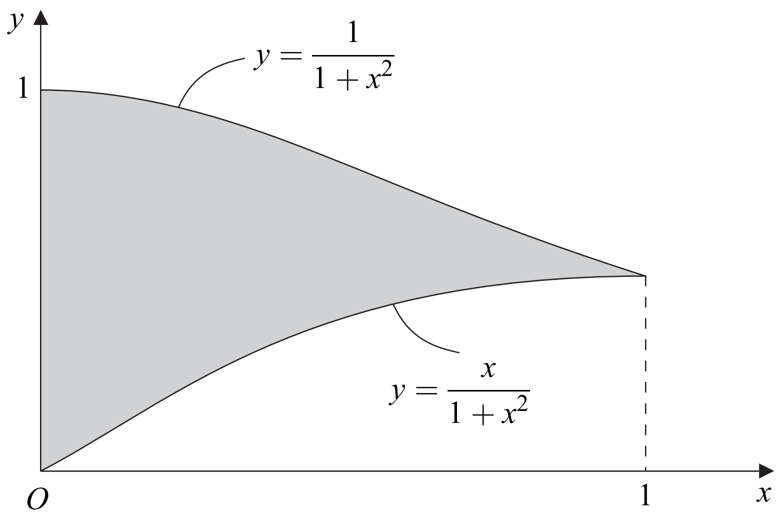
(ii) Hence find $\frac{dx}{dy}$ in terms of y. *(1 mark)*

(iii) Hence, using an appropriate trigonometrical identity, show that $\frac{dy}{dx} = \frac{1}{1 + x^2}$. *(2 marks)*

(iv) Evaluate $\int_0^1 \frac{1}{1 + x^2} \, dx$. *(2 marks)*

(c) The diagram shows the graphs of $y = \frac{1}{1 + x^2}$ and $y = \frac{x}{1 + x^2}$ for $0 \leqslant x \leqslant 1$.

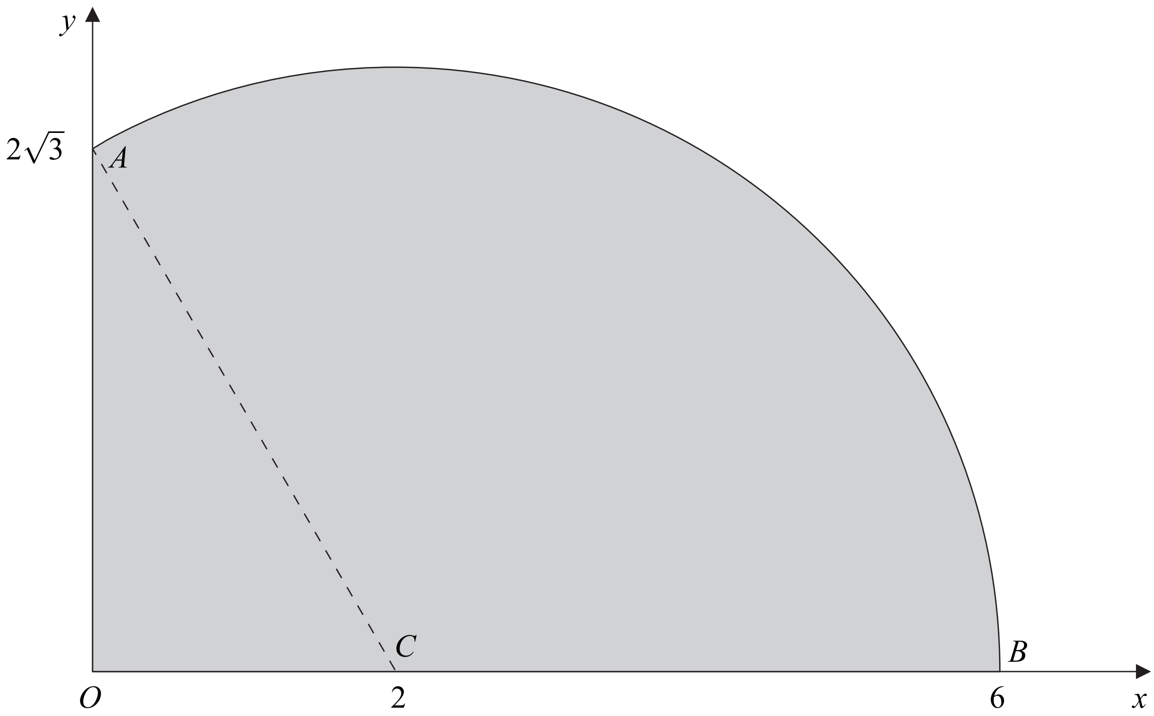
The graphs intersect at the point where $x = 1$.



Show that the area of the shaded region is $\frac{\pi}{4} - \frac{1}{2} \ln 2$. *(2 marks)*

5 The diagram shows the graph of $y = \sqrt{16 - (x-2)^2}$ for $x \geqslant 0$ and $y \geqslant 0$.

The points of intersection of the curve with the coordinate axes are $A(0, 2\sqrt{3})$ and $B(6, 0)$.



(a) Use the trapezium rule, with six strips, to estimate the area of the shaded region. Give your answer to one decimal place. *(4 marks)*

(b) The curve forms part of a circle with centre $C(2,0)$.

(i) Write down the radius of the circle. *(1 mark)*

(ii) Show that the angle ACB is 120°. *(2 marks)*

(c) (i) Find the area of the sector ACB. *(1 mark)*

(ii) Hence show that the area of the shaded region is $\dfrac{16\pi}{3} + 2\sqrt{3}$. *(2 marks)*

(d) The shaded region is rotated about the x-axis through 360° to form a solid of revolution.

Use integration to find the volume of this solid. *(4 marks)*

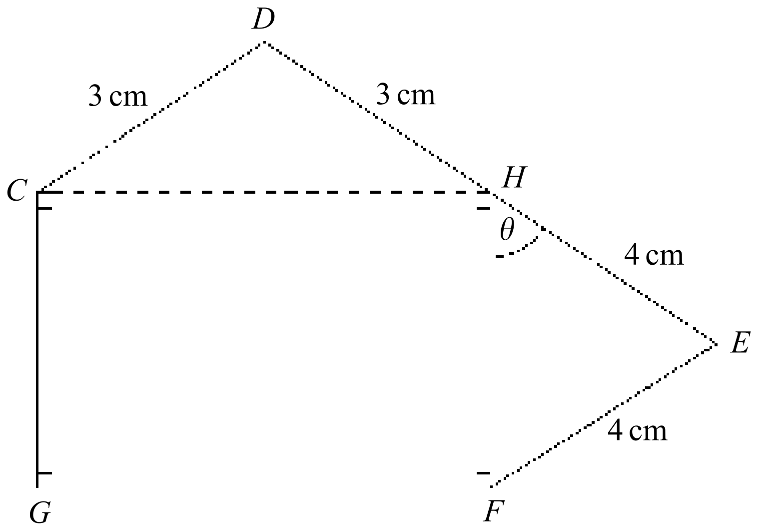
6 (a) Express

$$6 \sin \theta + 8 \cos \theta$$

in the form $R \sin(\theta + \alpha)$, where $R > 0$ and $\alpha < \frac{\pi}{2}$, giving the value of α to three decimal places. *(3 marks)*

(b) The diagram shows a pentagon *CDEFG*, formed by the rectangle *CHFG* and two isosceles triangles *CDH* and *EFH*, where *DHE* is a straight line.

The angle *EHF* is θ, $CD = DH = 3$ cm, $EH = EF = 4$ cm.



(i) The perimeter of the pentagon is P cm.

Show that $P = 14 + 6 \sin \theta + 8 \cos \theta$. *(3 marks)*

(ii) Write down the maximum possible value of P and find the value of θ at which this maximum value occurs. Give the value of θ to three decimal places. *(3 marks)*

(c) (i) The area of the pentagon is A cm^2.

Show that $A = 36.5 \sin 2\theta$. *(5 marks)*

(ii) Hence show that the perimeter of the pentagon with maximum area is $7(2 + \sqrt{2})$ cm. *(2 marks)*

END OF QUESTIONS

THERE ARE NO QUESTIONS PRINTED ON THIS PAGE

THERE ARE NO QUESTIONS PRINTED ON THIS PAGE

THERE ARE NO QUESTIONS PRINTED ON THIS PAGE