

General Certificate of Education  
January 2005  
Advanced Level Examination



**MATHEMATICS (SPECIFICATION A)**  
**Unit Mechanics 3**

**MAM3**

Monday 31 January 2005 Morning Session

**In addition to this paper you will require:**

- an 8-page answer book;
- the AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 20 minutes

**Instructions**

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MAM3.
- Answer **all** questions.
- Take  $g = 9.8 \text{ m s}^{-2}$  unless otherwise stated.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.
- Tie loosely any additional sheets you have used to the back of your answer book before handing it to the invigilator.

**Information**

- The maximum mark for this paper is 60.
- Mark allocations are shown in brackets.

**Advice**

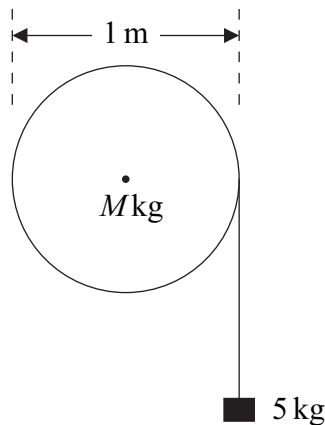
- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

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Answer **all** questions.

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- 1 (a) Show, by integration, that the moment of inertia of a uniform disc, of mass  $m$  and radius  $r$ , about an axis through its centre and perpendicular to its plane is  $\frac{1}{2}mr^2$ . (5 marks)
- (b) A particle of mass 5 kg hangs from the end of a light, inextensible cord which is wound tightly round the circumference of a wheel of diameter 1 metre and mass  $M$  kg. The wheel, which is modelled as a uniform disc, is free to rotate in a vertical plane about a fixed horizontal axis through its centre.

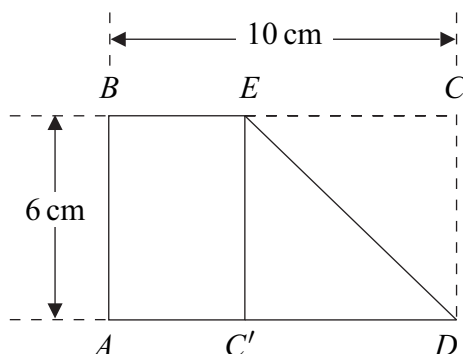


Show that the moment of inertia of the wheel about this axis is  $\frac{M}{8} \text{ kg m}^2$ . (1 mark)

- (c) The system is released from rest. When the particle has fallen a distance of 4 metres, its speed is  $8 \text{ m s}^{-1}$ .
- (i) Calculate the potential energy lost by the 5 kg particle when it has fallen 4 metres. (1 mark)
- (ii) Calculate the angular speed,  $\omega$ , of the wheel when the particle has fallen 4 metres. (1 mark)
- (iii) Find the mass,  $M$ , of the wheel. (5 marks)

- 2 A uniform rectangular lamina  $ABCD$  has sides  $AB = 6$  cm and  $BC = 10$  cm.

The section  $ECD$  is folded along the line  $DE$  so that  $DC$  lies along  $DA$  and the point  $C$  is at  $C'$ , as shown in the diagram.



- (a) (i) Show that the centre of mass of the triangular section  $EC'D$  of the new body is 6 cm from  $AB$ . (2 marks)
- (ii) Hence show that the centre of mass of the new body is 4.4 cm from  $AB$ . (2 marks)
- (iii) Find the distance of the centre of mass of the new body from  $AD$ . (2 marks)
- (b) The body is now freely suspended from  $B$  and is then rotated in the vertical plane of the body to the position where  $AD$  is horizontal, as in the diagram above. It is held in equilibrium in this position by a couple of magnitude  $K$ .

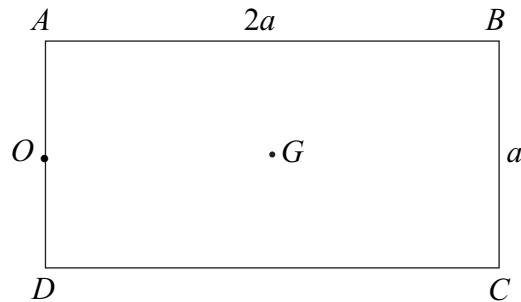
Given that the weight of the body is 2 N, find the value of  $K$ . (2 marks)

- 3 Forces  $4\mathbf{i} + 5\mathbf{j}$ ,  $2\mathbf{i} - \mathbf{j}$  and  $-3\mathbf{i} + 2\mathbf{j}$  act through the points  $(1, 2)$ ,  $(3, -1)$  and  $(-2, 1)$  respectively.

- (a) The three forces are equivalent to the single force  $\mathbf{P}$ . Show that  $\mathbf{P} = 3\mathbf{i} + 6\mathbf{j}$ . (1 mark)
- (b) (i) Show that the forces have a total moment of magnitude 5 about the origin. (4 marks)
- (ii) State the sense of this total moment. (1 mark)
- (c) The line of action of  $\mathbf{P}$  cuts the  $y$ -axis at the point  $(0, d)$ . Find the value of  $d$ . (3 marks)

Turn over ►

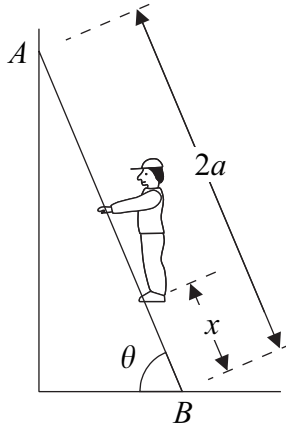
4 A uniform rectangular lamina  $ABCD$  has mass  $m$ , with  $AB = 2a$  and  $BC = a$ .



- (a) (i) Show that the moment of inertia of the lamina about an axis perpendicular to its plane and passing through its centre of mass,  $G$ , is  $\frac{5ma^2}{12}$ . (1 mark)
- (ii) Hence find the moment of inertia of the lamina about an axis perpendicular to its plane and passing through  $O$ , the midpoint of  $AD$ . (2 marks)
- (b) The lamina is free to rotate in a vertical plane about a fixed horizontal axis which is perpendicular to its plane and passes through  $O$ . Initially the lamina is held with  $OG$  horizontal. The lamina is then released. At time  $t$  after release,  $OG$  makes an angle  $\theta$  with the horizontal.
- (i) Show that  $\dot{\theta}^2 = \frac{24g \sin \theta}{17a}$ . (4 marks)
- (ii) Hence, or otherwise, find the value of  $\ddot{\theta}$  in terms of  $a$ ,  $g$  and  $\theta$ . (2 marks)
- (c) Find, in terms of  $m$ ,  $g$  and  $\theta$ :
- (i) the component of the force at  $O$ , in the direction  $GO$ , which the hinge exerts on the lamina; (3 marks)
- (ii) the component of the force at  $O$ , perpendicular to  $GO$ , which the hinge exerts on the lamina. (3 marks)
- (d) Find the value of  $\theta$  when the total reaction at  $O$  makes an angle of  $45^\circ$  with  $GO$ . (2 marks)

- 5 A uniform ladder  $AB$ , of weight  $W_1$  and length  $2a$ , rests on rough horizontal ground, with  $A$  against a smooth vertical wall. The ladder makes an angle  $\theta$  with the horizontal where  $\tan \theta = 3$ .

A man, of weight  $W_2$ , is standing at a distance  $x$  from the bottom of the ladder. The system is in equilibrium. Assume that the man's centre of mass is directly above his feet and that any contact forces through his hands are negligible.



- (a) Draw a diagram showing all the forces acting on the ladder. (2 marks)
- (b) Find, in terms of  $W_1$ ,  $W_2$ ,  $a$  and  $x$ , the force of the wall on the ladder at  $A$ . (4 marks)
- (c) Given that the coefficient of friction between the ladder and the ground is  $\frac{3}{10}$ , show that

$$x \leq \frac{a(4W_1 + 9W_2)}{5W_2}. \quad (4 \text{ marks})$$

- (d) Deduce that the man can stand at the top of the ladder without it slipping, provided that  $W_2 \leq 4W_1$ . (3 marks)

**END OF QUESTIONS**

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